Fiscal Externalities in the Open Economy

by

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I. Introduction

Since the late 1970s industrial economies have experienced massive cutbacks in public investment expenditures, all during periods of slow productivity growth, real resource scarcities, and record high government budgetary deficits. Some of this retrenchment might have been necessary to improve public sector efficiency or to deal with revenue shortfalls. In any event serious concerns have been raised about the consequences for future economic growth and welfare, and for a country's ability to compete internationally.

In Park (1995) I conduct an empirical study of public investment using OECD panel and individual country data. From an aggregate growth accounting framework, I find significant output elasticities for public infrastructural capital and public R&D capital. Moreover I find positive cross-country spillover effects associated with public R&D capital. Public investments are also found to stimulate private investment nationally and globally through spillovers. This gives some inkling of the costs, or forgone benefits, due to public investment reductions in the OECD - not to mention cutbacks in non-OECD economies.

This paper turns to a theoretical investigation of public investment in an open-economy, overlapping-generations (OLG) framework[1]. The short run and long run impacts on private consumption, international

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[1] I have chosen an OLG framework to avoid Ricardian equivalence between debt issue and taxation, and to avoid the restriction found in representative agent models that require the steady-state interest rate to equal the subjective time preference rate, or else steady-state consumption will not exist. Unless time preference rates are endogenous, the restriction fixes the steady-state stock of capital and output exogenously.

competitiveness, and current account behaviour are evaluated. The framework adopted is along the lines of previous work by Alogoskoufis-van der Ploeg (1991), Buiter (1987), Frenkel-Razin (1987), Giovannini (1988), and Obstfeld (1989). There are three main points of departure. First, I focus on productive government spending. In previous work the attention is on taxation and budgetary deficits; government spending is assumed not to affect private production or marginal utility. In models where government spending is unproductive, a permanent increase in government spending is a resource withdrawal and is tantamount to a reduction in the lifetime wealth of private agents. Wealth is further reduced if increased external debt or crowding out of private capital results. In the model of this paper, productive government spending boosts supply, stimulates private capital, raises lifetime human wealth and possibly financial wealth.

Secondly, previous work tends to stress "price" channels - such as the interest rate, terms of trade, real exchange rate - as the means by which fiscal policies in one country are transmitted to the rest of the world. Interdependence through prices generates "pecuniary externalities". This paper, however, stresses "technological" channels: public capital of one country serves as an input not only in the country's own production function but also in the production functions of other countries. This type of direct interdependence generates "technological externalities" (assuming there is no international compensation mechanism to internalize the external effects of public capital). The distinction between these two types of externalities is elaborated in Scitovsky (1954), and the relevance for the open-economy discussed in Buiter-Kletzer (1990). International interdependence through prices is not by itself a source of market failure, requiring international fiscal policy coordination. Indeed it reflects the market mechanism at work, provided every economic influence is transmitted through prices. Factor-use-related externalities of the sort I look at create direct dependence and do not operate through the market mechanism, and is therefore a source of divergence between private and social benefits. As Scitovsky (1954, p. 146) notes:

> "What is puzzling ... is that interdependence through the market mechanism should be held to account for the failure of the market economy to lead to the socially desirable optimum, when equilibrium theory comes to the opposite conclusion and relies on market interdependence to bring about an optimum situation."

A third point of departure is that I allow for endogenous government behaviour. In particular, governments can choose a stream of public investment to maximize a social welfare function, of which there are two kinds. The social welfare function for a national planner is defined over the welfare of agents alive and to-be-born in a nation. The global planner's social welfare function is a weighted average of national social welfare functions. While the command optimum under each type of planner is time-consistent, the key difference is that the global planner takes into account the cross-country spillovers from public capital.

An earlier paper on public investment in the open-economy is presented in Devereux (1987a). The author develops a two-period model with no private capital (the national stock of capital could be thought of as being entirely government-owned) and no installation costs to capital formation. The paper presents an example of spillovers occurring through price variables. For instance, in a Nash equilibrium a creditor country overinvests (relative to an internationally coordinated investment level) and a debtor country underinvests because an increase in public investment raises the world interest rate which benefits the creditor and hurts the debtor. A recent paper on "technological" spillovers is presented in Alogoskoufis-van der Ploeg (1991). The authors extend an endogenous growth model to a two-country, one-commodity setting. Endogenous growth is driven by externalities from private capital. It is assumed that the average stock of national private capital embodies 'knowledge'. International spillovers of knowledge are captured by entering each country's capital stock into each other's production function. Unfortunately I do not find empirical support for this type of production function. I fail to find the presence of external effects in the aggregate or aggregate per-capita stock of private capital. The authors also treat public expenditures as unproductive, and find that a higher global ratio of public consumption to GDP reduces world savings and growth.

The paper is organized as follows: the next section describes the model. Section III discusses endogenous government behaviour (Appendix I explains how government objective functions are derived). Section IV analyzes the small open-economy, and Section V the two-country version. Both steady-state and dynamic analyses are provided (Appendix II provides background detail on the dynamic simulations). Section VI summarizes the main findings and suggests extensions for further research.

II. Model

The following model describes a two-country world inhabited by utility-maximizing consumers and value-maximizing firms. Each country is specialized in the production of a tradable good, and agents desire the consumption of both the domestic and foreign good. A perfectly integrated international financial market exists in which a global bond (denominated in terms of the domestic good) is traded. Labour and goods markets clear continuously.

Governments in each country engage in public capital formation which not only enhances own private-sector productivity but also spills over to enhance private-sector productivity abroad. Public capital formation is broadly defined here to cover not only physical capital (such as social infrastructure, telecommunications, transportation) but also intangible capital (such as public R&D capital). Governments can only tax and borrow from their own residents.

I will focus on developing the home country. The foreign economy is developed in an analogous manner and is summarized in Table 1. (Foreign variables are denoted by asterisks.)

Production Sector:

The production function is assumed to be Cobb-Douglas over private inputs K capital, and L labour:

$$(1)\quad Y = A\,F(K, L) = A\,K^{\beta_k}\,L^{\beta_L}, \qquad 0 < \beta_k, \beta_L < 1$$

I specify the total factor productivity term as: $A = \pi\, G^{\beta_G}\, G^{*\,\beta_{G^*}}$ where π is a constant to which we assign all omitted variables, and where G and G* are the stocks of domestic and foreign public capital respectively. They are assumed to be imperfectly substitutable and essential.

The production function can be rewritten as:

$$(1)' \quad Y = F(G, G^*, K, L) = \pi\, G^{\beta G}\, G^{*\,\beta G^*}\, K^{\beta k}\, L^{\beta L}, \quad \pi > 0$$

It is still necessary to decide upon the "returns to scale". I examine three possibilities:

$\beta_G + \beta_{G^*} + \beta_K + \beta_L = 1$ CRTS across all inputs

$\beta_G + \beta_K + \beta_L = 1$ CRTS across domestic inputs

$\beta_K + \beta_L = 1$ CRTS across private inputs[1],

where CRTS denotes "constant returns to scale".

For simplicity I assume that labour supplies are inelastic and normalized to unity ($L = L^* = 1$). Firms are identical price-takers and face quadratic internal costs of adjustment. The firm's valuation functional is[2]:

$$(2) \quad \max \; V(t) = \int_t^{\infty} \{(1 - \tau_k)(F(.) - w(s)L(s)) - J_p(s)\}\, e^{-\int_t^s r(u)du}\, ds$$

$$\text{where } J_p(s) = I_p(s) + \frac{\psi I_p(s)^2}{2\,K(s)}$$

$$\text{and } \dot{K}(s) = I_p(s) - \delta\, K(s)$$

and where δ is the geometric depreciation rate, $\psi > 0$ the adjustment cost parameter, and τ_k the capital taxation rate. The firm maximizes $V(t)$ by

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1 In my empirical study of OECD economies I tend to find support for CRTS across private K and L.

2 To avoid cluttering up the notation I omit time subscripts in the production function F(.) and in the marginal products, such as $F_K(.)$, $F_G(.)$.

choosing L(s) and K(s) for all s $\in$ [t,∞). F(.) is the production function, r the domestic interest rate, w the wage rate, and J_p is the sum of gross private investment I_p and output used up in transforming goods into capital. K is thus the net of depreciation stock of private capital. Only home output is used for domestic capital formation.

The solutions to the firm's maximization are:

$$\text{(3a)}\quad w(s) = F(.) - K(s)\, F_K(.)$$

$$\text{(3b)}\quad q_p(s) = 1 + \psi\left(\frac{I_p(s)}{K(s)}\right) \equiv \frac{\partial J_p(s)}{\partial I_p(s)}$$

$$\text{(3c)}\quad r(s) = \frac{\dot{q}_p(s)}{q_p(s)} + \frac{(1-\tau_k)F_K(.) + \frac{\psi}{2}\left(\frac{I_p(s)}{K(s)}\right)^2}{q_p(s)} - \delta$$

for all s $\in$ [t,∞). F_K is the marginal product of capital. Condition (3a) is the familiar marginal product of labour equated to the competitive wage. Condition (3b) is similar; the shadow price of capital, q_p, is equal to its replacement cost. Condition (3c) states that the value of the marginal product of capital (ie. the sum of the after-tax marginal product of capital in production and the marginal contribution of capital in raising the productivity of investment in installation) equals the cost of capital (ie. the sum of the interest rate and depreciation rate less capital gains).

Rewriting (3c) as:

$$\text{(3c)'}\quad \dot{q}_p(s) = (r(s) + \delta)\, q_p(s) - [(1-\tau_k)F_K(.) + \frac{\psi}{2}\left(\frac{I_p(s)}{K(s)}\right)^2],$$

multiplying both sides by K(s) and solving forward (and imposing a no-bubble condition[1]) gives

$$\text{(3d)}\quad q_p(t)K(t) = \int_t^{\infty} e^{-\int_t^s r(u)du}[(1 - \tau_k)F_K K(s) - J_p(s)]\, ds.$$

Assume further that there are constant returns to scale across private inputs so that when K, L are paid their marginal products, aggregate output is fully exhausted - ie. $F(.) = F_L.L + F_K.K$. Substituting this relationship into (3d) gives $q_p(t)K(t) = V(t)$, or

$$\text{(3d)'}\quad q_p(t) = \frac{V(t)}{K(t)}$$

That is, the value of the private capital stock equals the value of firms, which in turn equals the present discounted sum of the future marginal value products of private capital.

Consumer Sector:

The economy is populated by overlapping generations of agents whose lifespans are uncertain[2]. The birth and death rates are both given by λ as is the size of cohorts born at every instant. This suitable normalization yields a population size of unity at all times:

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ie. $\lim_{s \Rightarrow \infty} K(s)q_p(s)\, e^{-\int_t^s r(u)du} = 0$

2 The approach is based on Blanchard (1985).

ie. $\int_{-\infty}^{t} \lambda\, e^{-\lambda(t-s)} ds = 1$

All agents earn the same wage rate and pay the same lump sum taxes. The rest of this section is divided into two parts: the first part will describe an individual agent's consumption and savings decisions; the second part will aggregate across individuals.

Individual Consumer Behavior

For each variable x(t,v), t indexes time and v the vintage of the agent - ie. the time of birth ($v \leq t$). The individual agent allocates financial wealth among three perfectly substitutable assets - government bonds, international bonds, and claims on the stock of private home country capital (equities) - each yielding a rate of return r(t). The government and international bonds are of the fixed market value and variable interest rate type while equities have variable market value, q_p, as derived earlier.

The individual derives utility homothetically from the domestic and foreign good, faces a constant probability of death, and has rational, single-valued expectations. The individual's intertemporal objective functional is thus:

$$(4)\quad U(t,v) = \int_{t}^{\infty} u(\tilde{c}(s,v))\, e^{-(\rho + \lambda)(s - t)} ds$$

where ρ is the time preference rate and

(4a) $u(\tilde{c}(s,v)) = \dfrac{\tilde{c}(s,v)^{1-\sigma}}{1-\sigma}$ for $\sigma \neq 1$

(4b) $u(\tilde{c}(s,v)) = \log(\tilde{c}(s,v))$ for $\sigma = 1$

(4c) $\tilde{c}(s,v) = c_H(s,v)^{\alpha}\, c_F(s,v)^{1-\alpha}$

c_H is the consumption of domestic output
c_F is the consumption of foreign output

where $\frac{1}{\sigma}$ is the intertemporal elasticity of substitution and α is the share of the consumer's expenditure on domestic output.

The individual maximizes (4) subject to the following flow budget constraint for each time $s \in [t,\infty)$:

(5) $\dot{a}(s,v) = (r(s) + \lambda)\, a(s,v) + w(s,v) - \tau(s,v) - (1 + \tau_c)\, c(s,v)$

where

(5a) $a(s,v) = b(s,v) + z(s,v) + q_p(s)k(s,v)$

(5b) $c(s,v) = c_H(s,v) + p(s)\, c_F(s,v)$

(5c) $c_H(s,v) = \alpha\, c(s,v)$

(5d) $p(s)c_F(s,v) = (1-\alpha)\, c(s,v)$

(5c) and (5d) are obtained by solving the intratemporal problem of maximizing (4c) subject to (5b). $c(s,v)$ represents total consumption expenditures, p the relative price of the foreign good in terms of the domestic good, τ lump-sum taxes (age-independent), τ_c the consumption tax rate, b holdings of government bonds, and z holdings of the internationally traded bond. The term $\lambda a(s,v)$ reflects the presence of a perfectly competitive annuity market. Agents are assumed not to leave bequests. Instead each agent contracts with an annuity company to transfer all her asset holdings a(s,v) to the company upon accidental death, and in return to earn interest on her asset holdings while alive - say i(s)a(s,v) where i(s) is the rate of interest. Since λ people die each instant, the annuity firm receives λa(s,v) and pays out i(s)a(s,v) each instant. Perfect competition in the annuity market leads to zero profits so that $\lambda = i(s)$.

Maximizing (4) subject to (5), after substituting (5c,d) into (4c), yields:

$$\text{(6a)} \quad \frac{\dot{c}(s,v)}{c(s,v)} = \frac{1}{\sigma}\left(r(s) - \rho + (1-\alpha)(1-\sigma)\frac{\dot{p}(s)}{p(s)}\right) \quad \text{for } s \in [t, \infty)$$

which describes the optimal evolution of consumption for the individual agent[1]. The term inside the brackets represents the rate of interest in terms of the domestic consumption basket. A rising $\frac{\dot{p}(s)}{p(s)}$ represents a declining terms of trade. The reaction of the individual to it depends on the intertemporal elasticity parameter times the share of imported goods in her consumption; if for example, $\sigma > 1$, the adverse wealth effect dominates and the consumer consumes less relative to the future. If $\sigma < 1$ the substitution effect dominates and the consumer consumes more relative to the future in anticipation of higher future prices.

Rolling the budget constraint forward[2] and substituting (6a) into it gives:

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[1] The consumption tax rate drops out since it is assumed to be a time-invariant exogenous policy instrument. As a result no intertemporal distortion is introduced in the time profile of the individual's consumption expenditures.

[2] Imposing also the transversality condition:

(6b) $c(t,v) = \theta(t)\,[a(t,v) + h(t,v)]$

where

(6c) $$\theta(t) = \left\{ \int_t^{\infty} e^{\int_t^s \frac{1}{\sigma}\left((1-\sigma)\,(r(u) + (\alpha-1)\frac{\dot{p}(s)}{p(s)}) - \rho - \lambda\sigma\right)du} ds \right\}^{-1}$$

(6d) $$h(t,v) = \int_t^{\infty} e^{-\int_t^s (\,r(u) + \lambda)\,du}\,[w(s,v) - \tau(s,v)]\;\; ds$$

Consumption is thus a linear function of the present value of human h(t,v) and non-human wealth a(t,v). The discount factor for human wealth effectively incorporates the probability of death. $\theta(t)$ is the marginal propensity to consume out of total wealth; note that $\frac{\partial\theta(t)}{\partial r(t)} \geq 0$ as $\frac{1}{\sigma} \leq 1$ and vice versa.

Aggregate Consumer Behavior

For each individual variable x(t,v), the aggregate counterpart x(t) can be obtained by aggregating across all cohorts alive:

$$x(t) = \int_{-\infty}^{t} x(t,v)\lambda e^{-\lambda(t-v)} dv$$

Applying this procedure provides the following:

$$\lim_{s\Rightarrow\infty} a(s,v)e^{-\int_t^s (r(u) + \lambda\,)du} = 0$$

$$\text{(7a)}\quad (1 + \tau_c)\, c(t) = \theta(t)\,[\, a(t) + h(t)]$$

$$\text{(7b)}\quad c(t) = c_H(t) + p(t)\, c_F(t) \quad \text{where } c_H(t) = \alpha\, c(t) \text{ and}$$

$$p(t)\, c_F(t) = (1\ \alpha)c(t)$$

$$\text{(7c)}\quad a(t) = b(t) + z(t) + q_p(t)k(t)$$

$$\text{(7d)}\quad h(t) = \int_t^{\infty} e^{-\int_t^s (r(u) + \lambda)\, du}\, [w(s) - \tau(s)]\, ds$$

$$\text{(7e)}\quad \theta(t) = \left\{ \int_t^{\infty} e^{\int_t^s \frac{1}{\sigma}\left((1-\sigma)\,(r(u) + (\alpha-1)\frac{\dot{p}(s)}{p(s)}) - \rho - \lambda\sigma\right)du}\, ds \right\}^{-1}$$

The dynamical counterparts are:

$$\text{(8a)}\quad (1 + \tau_c)\, \dot{c}(t) = \frac{1}{\sigma}\left(r(t) - \rho + (1-\alpha)(1-\sigma)\frac{\dot{p}(t)}{p(t)} \right)(1 + \tau_c)\, c(t) - \lambda\, \theta(t)\, a(t)$$

$$\text{(8b)}\quad \dot{\theta}(t) = \theta(t)^2 + \theta(t)\left(\frac{1}{\sigma}\{(1-\sigma)[r(t) + (1-\alpha)\frac{\dot{p}(t)}{p(t)}] - \rho - \sigma\lambda\} \right)$$

$$\text{(8c)}\quad \dot{h}(t) = (r(t) + \lambda)\, h(t) + \tau(t) - w(t)$$

$$\text{(8d)}\quad \dot{a}(t) = r(t)\, a(t) + w(t) - \tau(t) - (1 + \tau_c)\, c(t)$$

In deriving h(t) it is assumed that wages and lump-sum taxes are vintage independent. In deriving a(t) it is assumed that newly born agents possess no financial wealth, ie. a(t,t) = 0. In (8a) it is seen that if $\lambda = 0$ the standard Euler equation for an infinitely-lived representative agent is obtained. Otherwise aggregate consumption will differ from that in a representative agent economy by $\lambda\theta(t)a(t)$ - ie. since the newly born agents have no financial wealth, one must deduct the marginal propensity to consume times the level of financial wealth ($\theta(t)a(t)$) from each of the λ new agents. In (8d) note that the $\lambda a(t)$ interest income term cancels in the aggregate. The equal birth and death rate of agents simply results in $\lambda a(t)$ being transferred from the deceased population to the living.

Government Sector

In this subsection I outline the basic government budget constraint. The next section derives behavioral specifications for the government.

Government spending here includes public investment i_g and unproductive government consumption expenditures c_g such as those purchases necessary to maintain the system of government but which have otherwise no direct effect on private sector production or utility.[1] Government revenues come from lump-sum taxes, capital income taxes, and consumption taxes imposed on domestic agents. Under certain circumstances (to be discussed in the next section) the government can also obtain revenues by charging a user fee for its stock of capital G or allow private firms to rent G. Finally the government can also borrow from the domestic private sector. Let b(t) denote the stock of government debt. The government budget constraint is thus:

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[1] In order to isolate the impacts of "productive" public investment, I abstract from other public goods spending that may impinge on private sector productivity or utility directly. Here c_g is treated exogenously. By treating it this way, I can better differentiate the impacts of G versus c_g. The way I treat c_g is the way government spending is treated in much of the existing literature.

(9) $\dot{b}(t) = r(t)\, b(t) + J_g(t) + c_g(t) - \tau(t) - \tau_c c(t) - \tau_k F_K K(t) - \xi(t)G(t)$

where

$$J_g(t) = I_g(t) + \frac{\psi I_g(s)^2}{2\, G(s)}$$

and $\dot{G}(s) = I_g(s) - \delta\, G(s)$

I assume that there are also adjustment costs to installing public capital. For simplicity I assume that the adjustment cost parameter and depreciation rate are identical to those for private capital. Again only home output can be used to augment the stock of domestic public capital[1]. The user fee for public capital is given by $\xi(t)$.

Solving (9) forward yields the intertemporal government budget constraint[2]:

(9a) $$\int_t^{\infty} e^{-\int_t^s r(u)du} [\tau(s) + \tau_c c(s) + \tau_k F_K K(s) + \xi(s)G(s)]\, ds$$

$$= \int_t^{\infty} e^{-\int_t^s r(u)du} [J_g(s) + c_g(s)]\, ds + b(t)$$

The LHS is the present discounted value of future revenues; the RHS is the present value of future spending plans plus initial debt. In order to help ensure long run public sector solvency I will be adopting the following feedback rule for lump-sum taxation:

[1] Similarly, only foreign output can be used to augment the stock of foreign public and private capital.
[2] After imposing the transversality condition:

$$\lim_{s \Rightarrow \infty} b(s) e^{-\int_t^s r(u)du} = 0$$

(9b) $\tau(t) = \tau_0 + \eta\dot{b}(t) \qquad \eta < -1$

where τ_0 is the exogenous component of lump sum taxes.

Linkages with the Foreign Sector

There is an internationally traded bond denominated in terms of domestic output, denoted by z, that is held by wealth holders in both countries. If $z > 0$ domestic agents are net lenders to the rest of the world, and net borrowers if $z < 0$. On net, the global capital market clears:

(10a) $z(t) + p(t)z^*(t) = 0$

(10b) $r(t) = r^*(t) + \dfrac{\dot{p}(t)}{p(t)}$

where asterisks denote foreign country variables. (10b) is the uncovered real interest parity condition.

The home country's trade balance is $TB(t) = c_H^*(t) - p(t)c_F(t) = \alpha^* p(t)c^*(t) - (1 - \alpha)c(t)$, where α^* is the share of foreign consumption expenditures on the domestically produced good. The home balance of payments identity is:

(11) $\dot{z}(t) = r(t)\, z(t) + TB(t)$

The LHS is minus the capital account balance; the RHS is the current account balance (sum of the service account balance $r(t)z(t)$ and the trade balance). The demand for domestic output y(t) comes from domestic and foreign consumers and domestic investors:

$$y(t) = c_H(t) + c_H{}^*(t) + c_g(t) + J_p(t) + J_g(t)$$

$$\text{or } y(t) = \alpha c(t) + (1 - \alpha)c^*(t) + c_g(t) + J_p(t) + J_g(t)$$

Substituting this into (11), using the TB(t) definition, provides another way to express the home country's balance of payments identity:

$$(11)' \quad \dot{z}(t) = r(t)\, z(t) + y(t) - c(t) - c_g(t) - J_p(t) - J_g(t)$$

I also note for future reference the foreign economy's balance of payments identity and goods market-clearing condition:

$$(11)'' \quad \dot{z}^*(t) = r^*(t)\, z^*(t) + y^*(t) - c^*(t) - c^*{}_g(t) - J^*{}_p(t) + J^*{}_g(t)$$

$$\text{where } y^*(t) = c_F(t) + c_F{}^*(t) + c^*{}_g(t) + J^*{}_p(t) + J^*{}_g(t)$$

$$\text{or } y^*(t) = (1 - \alpha)\frac{c(t)}{p(t)} + (1 - \alpha)c^*(t) + c^*{}_g(t) + J^*{}_p(t) + J^*{}_g(t)$$

III. Endogenous Government Behavior

In the previous section public investment (i_g) and consumption (c_g) were exogenous. Public capital formation will now be chosen optimally.

There is a shadow price, q_g, associated with public capital which equals the present discounted value of its future marginal products. I will first characterize the optimum conditions and discuss alternative ways to charge for the services of public capital. There are two types of planners: national and global. The national planner chooses a public investment strategy to maximize national welfare while the global planner chooses public investments in each country to maximize world welfare.

I will continue to leave c_g exogenous. It seems to be a good benchmark for comparing the open-economy effects of productive versus unproductive government expenditure. As indicated earlier, the treatment of government spending in much of the previous literature is similar to the way c_g is treated here. I will also abstract from issues of optimal taxation. Government behavior is optimal as far as its choice of investment is concerned. However it is not a fully optimizing government because its choice of financing mix need not be optimal. If the government were fully optimizing and no distortionary taxes were needed, the planner would choose public capital formation optimally and carry out a system of intergenerational and intragenerational redistributions of lump-sum taxes (age-dependent, or age-independent with debt policy) such that at the margin the planner had no further incentive to shift one unit of resource between periods and/or between generations alive contemporaneously. That is, if fiscal policy were fully optimal (and instruments fully available) there should be no need to alter the fiscal policies set, except in circumstances of unexpected shocks, since the fiscal policies in place would be Pareto-optimal intergenerationally and intragenerationally. For this reason, I assume that governments are not fully optimizing over all their instruments (in particular, those related to financing) so that exogenous variations in fiscal instruments such as lump-sum taxes or distortionary taxes can be studied[1].

I assume governments are non-paternalistic, utilitarian planners. Appendix I outlines how individual welfare functions in an OLG model can be aggregated. The home-country national planner maximizes (with respect to choices of K, $\dot{K}$, G, $\dot{G}$, z, $\dot{z}$, and c):

$$(12)\quad SW_t = \int_t^{\infty} u(c(s))\, e^{-\gamma(s-t)} ds$$

(where γ is the planner's time preference rate) subject to the national resource constraint:

$$(11)'\quad \dot{z}(s) = r(s)\, z(s) + y(s) - c(s) - c_g(s) - J_p(s) - J_g(s), \text{ for all } s \in [t,\infty).$$

Assume there is no need for distortionary taxation - ie. $\tau_K = \tau_c = 0$ for all s. The necessary conditions are for $s \in [t,\infty)$:

$$(13a)\quad \frac{\dot{c}(s)}{c(s)} = \frac{1}{\sigma}[r(s) + (\alpha-1)(1-\sigma)\frac{\dot{p}(s)}{p(s)} - \gamma]$$

$$(13b)\quad \frac{\dot{q}_p(s)}{q_p(s)} + \frac{F_K + \frac{\psi}{2}\left(\frac{i_p(s)}{K(s)}\right)^2}{q_p(s)} - \delta = \frac{\dot{q}_g(s)}{q_g(s)} + \frac{F_G + \frac{\psi}{2}\left(\frac{i_g(s)}{G(s)}\right)^2}{q_g(s)} - \delta = r(s)$$

where

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[1] It would be desirable to introduce different types of uncertainty and observe how a fully optimizing planner responds, and examine what kinds of optimal policy rules emerge in the presence of uncertainty.

$$(13c)\ q_p(s) = 1 + \psi\left(\frac{i_p(s)}{K(s)}\right)$$

$$(13d)\ q_g(s) = 1 + \psi\left(\frac{i_g(s)}{G(s)}\right).$$

In steady state, $q_p = q_g = 1 + \psi\delta$, which is not equal to one because there is a constant amount of steady state investment (namely $i_g = \delta G$, $i_p = \delta K$). The two q's are also equal in steady-state because it costs the same to replace the two types of capital[1]. The national planner chooses public capital in such a way that the returns (capital gains plus the marginal productivity terms less depreciation) to the two types of capital are equal, and equated to the interest rate (ie. intertemporal terms of trade). Aggregate consumption evolves according to the discrepancy between the real interest rate (in terms of the domestic consumption basket) and the planner's time preference rate. The long run interest rate equals the planner's time preference rate.

The global planner maximizes (with respect to K, $\dot{K}$, G, $\dot{G}$, K*, $\dot{K}^*$, G*, $\dot{G}^*$, c, and c*):

$$(14)\quad GW_t = \int_t^{\infty} [\, u(c(s)) + u^*(c^*(s))\,]e^{-\gamma'(s-t)}ds$$

subject to the world resource constraint:

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[1] In a linearized model where it is only valid to study small changes around the neighbourhood of steady-state, the two q's tend to be equal even out of steady-state.

$$\text{(14a)}\quad y(s) + p(s)y^*(s) = c(s) + p(s)c^*(s) + c_g(s) + c_g^*(s) + J_p(s) + p(s)J_p^*(s) + J_g(s) + p(s)J_g^*(s)$$

for $s \in [t,\infty)$.

The global planner is assumed to weight home and foreign welfare equally and apply the same time preference rate γ' to both countries. Let $\mu(s)$ be the shadow value of the world resource constraint. Necessary conditions for an optimum are:

$$\text{(15a)}\quad \frac{\dot{\mu}(s)}{\mu(s)} = (\alpha-1)(1-\sigma)\frac{\dot{p}(s)}{p(s)} - \sigma\frac{\dot{c}(s)}{c(s)} = (\alpha^*(1-\sigma^*) - 1)\frac{\dot{p}(s)}{p(s)} - \sigma^*\frac{\dot{c}^*(s)}{c^*(s)}$$

$$\text{(15b)}\quad -\frac{\dot{\mu}(s)}{\mu(s)} = \frac{\dot{q}_g(s)}{q_g(s)} + \frac{F_G + p(s)F^*_G + \frac{\psi}{2}\left(\frac{i_g(s)}{G(s)}\right)^2}{q_g(s)} - \gamma' - \delta$$

$$\text{(15c)}\quad -\frac{\dot{\mu}(s)}{\mu(s)} = \frac{\dot{q}_p(s)}{q_p(s)} + \frac{F_K + \frac{\psi}{2}\left(\frac{i_p(s)}{K(s)}\right)^2}{q_p(s)} - \gamma' - \delta$$

$$\text{(15d)}\quad -\frac{\dot{\mu}(s)}{\mu(s)} - \frac{\dot{p}(s)}{p(s)} = \frac{\dot{q}^*_g(s)}{q^*_g(s)} + \frac{F_{G*} + p(s)F^*_{G*} + \frac{\psi^*}{2}\left(\frac{i^*_g(s)}{G^*(s)}\right)^2}{q^*_g(s)} - \gamma' - \delta^*$$

$$\text{(15e)}\quad -\frac{\dot{\mu}(s)}{\mu(s)} - \frac{\dot{p}(s)}{p(s)} = \frac{\dot{q}^*_p(s)}{q^*_p(s)} + \frac{F^*_{K*} + \frac{\psi^*}{2}\left(\frac{i^*_p(s)}{K^*(s)}\right)^2}{q^*_p(s)} - \gamma' - \delta^*$$

where q_p, q_g are as defined before. Analogously,

$$(15f)\ q^*{}_p(s) = 1 + \psi^* \left(\frac{i^*{}_p(s)}{K^*(s)}\right)$$

$$(15g)\ q^*{}_g(s) = 1 + \psi^* \left(\frac{i^*{}_g(s)}{G^*(s)}\right)$$

The interpretation of (15a) is that $-\frac{\dot{\mu}}{\mu}$ plus the planner's time preference rate represent the return to consumption[1]. The returns to each of the four stocks of capital are equated to this return to consumption (denominated in terms of home country output). This also implies that the returns (capital gains plus the marginal productivity terms less depreciation) from each type of capital are equated.

To summarize, (13a-b) constitute consumption and production efficiency conditions for a national planner and (15a-e) the same for a global planner. A key difference is that the global planner takes into account the contribution each country's public capital stock makes to the other's production possibilities. These conditions turn public capital formation into an endogenous "policy rule".

There are special cases of (15a) that are worth noting - namely when any one of the following three holds: (i) $p = 1$; (ii) $\sigma = 1$; (iii) $\alpha = \alpha^*$. Each leads to:

$c = pc^*$

This simplifies the global model as the global planner treats the two countries as essentially two identical economies, so that $K=pK^*$, $G=pG^*$,

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[1] In a decentralized economy this return equals the competitive interest rate.

and y=py* and so forth, where p is the slope of the global production possibilities frontier. The case p=1 can be justified if the production functions are identical and global opportunity costs are constant (ie. equal ratios of marginal products among the factors of production in each country); $\sigma=1$ if the intertemporal substitution and wealth effects on consumption of intertemporal price changes cancel, and $\sigma=1$ causes the evolution of consumption to be independent of intratemporal price changes as well; finally, $\alpha=\alpha^*$ implies basically a one commodity world - domestic agents have the same propensity to consume home output as foreign agents have to consume home output.

I turn now to issues related to the valuation of public capital. I consider the national and global planners in turn.

National Planner

First, from (13b):

$$(13b)' \quad \dot{q}_g(t) = (r(s) + \delta)\, q_g(t) \; - (F_G + \frac{\psi}{2}\left(\frac{i_g(t)}{G(t)}\right)^2)$$

$$\text{or } (16) \quad q_g(t)G(t) = \int_t^{\infty} e^{-\int_t^s r(u)du} [F_G G(s) \; - J_g(s)]ds$$

As long as there are constant returns across private inputs - ie. $F = F_K K + F_L L$ - it is not possible to impute a convenient market share of output to government capital. Neither can it be assumed that $F_G = 0$ since that would contradict the assumption that public capital is productive. In the case where public capital cannot be priced or valued in the market I assume that it is provided "free" to the private sector despite a non-zero

F_G. The shadow price q_g remains unobservable (known only to the planner) as there is no market counterpart to it as there is for q_p.

If however there are less than constant returns across private inputs, total national output will not be fully exhausted. There are two possible ways the government can exact its share of aggregate output: it can charge a user fee for the services of public capital or lend its stock to private firms. This last scheme is analogous to the way agents in the model who own the stock of private capital lend their capital (q_pK) to private firms and earn a market return on it (rq_pK). I will indicate under what conditions the two schemes are equivalent.

To illustrate I rewrite the flow government budget constraint as:

$$(17)\ \dot{a}_g(t) = r(t)a_g(t) + \tau(t) - c_g(t)$$

where

$$(17a)\ a_g(t) = q_g(t)G(t) - b(t)$$

$a_g(t)$ represents the explicit net worth of the government (assets less liabilities)[1]. The LHS of (17) represents a net debit on the government's capital account balance while the RHS represents a net credit on the government's current account balance.

(i) User Fee Approach

Substituting (17a) and the time-differentiation of (16) into (17) gives:

[1] ie. not including the implicit assets represented by the present value of future taxes.

$$(18)\ \dot{b}(t) = r(t)b(t) + J_g(t) + c_g(t) - \tau(t) - F_G G(t)$$

From (18) an appropriate user fee for government capital would be $\xi(t) = F_G$. Then (18) can be rewritten as:

$$(18)'\ \dot{b}(t) = r(t)b(t) + J_g(t) + c_g(t) - \tau(t) - \xi(t)G(t)$$

The value-maximizing firm's objective functional should also be changed to:

$$(19)\quad V(t) = \int_t^{\infty} e^{-\int_t^s r(u)du}[F(.) - w(s) - J_p(s) - \xi(s)G(s)]ds$$

but the first-order conditions would be as before (since firms treat the stock of G as exogenously given).

The optimizing government would charge users the marginal product of its capital. Any other government might charge an $\xi(t)$ differing from the 'true' F_G or keep $\xi(t)$ fixed at some ξ_0. However, unless $\xi(t) = F_G$, q_pK will not equal the value of firms V. Recall from

$$(3d)\quad q_p(t)K(t) = \int_t^{\infty} e^{-\int_t^s r(u)du}[F_K K(s) - J_p(s)]ds$$

that the RHS equals V only if $F = F_K K + F_L L + F_G G = F_K K + wL + \xi G$, where $w = F_L$. In the event that $F > F_K K + F_L L + F_G G$, say due to the contribution of foreign public capital (ie. $F = F_K K + F_L L + F_G G + F_{G*}G*$), the home government is assumed to overcharge; ie. it charges $\xi(t) = F_G + F_{G*}(\frac{G*}{G}) > F_G$. Conversely if $F < F_K K + F_L L + F_G G$, the government undercharges for the use of its capital.

(ii) Rental Approach

The government under this approach lends its stock of capital to the private sector. The value functional is rewritten as:

$$(20)\quad V(t) = \int_t^{\infty} e^{-\int_t^s r(u)du} [F(.) - w(s) - J_p(s) - J_g(s)]ds$$

Firms now choose G and K to maximize value. A price q_p is paid to the owners of K, namely households, and q_g to the owner of G, the government. By the arbitrage condition (13b), K and G both earn the same market rate of return. If $F = F_K K + F_L L + F_G G$, then (3d) and (16) can be substituted into (20) to give:

$$(20)'\quad V = V_K + V_G, \text{ where } V_K = q_p K \text{ and } V_G = q_g G$$

that is, the value of a firm is divided between the private owners and the public sector.

It is possible (but not for certain) that $q_p = q_g$ even out of steady-state since both G and K come from the same good, face the same installation costs and depreciation rates. In addition firms may develop criteria which lead them to select G, K optimally only if these prices are equal. Again if $F > F_K K + F_L L + F_G G$, the gap being attributable to $F_{G*}G^*$, the price q_g will be greater than what (16) dictates. Firms are assumed to perceive the marginal product of domestic capital to be F_G' where $F_G'G = [F - F_K K - F_L L]$, the residual from total product after deducting private factor shares. The new price of public capital becomes $q_g' = q_g(1+v)$ where v, a markup, is obtained from $F_G'G = F_G G(1+v)$ and

F_G is the true marginal product[1]. Similarly if $F < F_K K + F_L L + F_G G$, the market q_g will be underpriced and ν will be a markdown..

Note that the user fee and rental approaches are equivalent when

$$\xi(t) = F_G = \frac{r(t)q_g(t)G(t) - (q_g(t)\dot{G}(t)) + J_g}{G(t)} .$$

The F_G expression on the right is just the time-differentiation of (16). The numerator is the rental earned less capital gains plus gross investment (including installation). Substituting the above into (18)', the government budget constraint under the user fee approach, gives (17), the net worth identity for the government. In other words the government can charge a user fee equal to what it would have earned had it lent its capital to the private sector. The value of a firm is $V_K = q_p K$. Through appropriate fees, the government can siphon its 'implicit' market share of the value of firms (ie. $V_G = V - V_K$).

---

[1] The ν comes from revising (16) to

$$(16)' \quad q_g(t)'G(t) = \int_t^{\infty} e^{-\int_t^s r(u)du} [F_G' - J_g(s)]ds$$

which gives:

$$\frac{q_g(t)'G(t)}{1+\nu} = \int_t^{\infty} e^{-\int_t^s r(u)du} [F_G - \frac{J_g(s)}{1+\nu}]ds$$

Comparing the LHS of the above to the LHS of the original (16), we find the new price to be $q_g' = q_g(1+\nu)$. Moreover $J_g'(s) = [J_g(s)/(1+\nu)]$; that is, it is necessary to revise Jg as well because by thinking that domestic public capital actually accounts for a fraction $(F_G G + F_{G*}G^*)/F$ of national output will overstate the actual size of the stock of G. Thus firms value public capital according to (16)' above, not (16), and by comparing (16) and (16)' we can determine the extent to which q_g' and J_g' differ from the 'true' q_g and J_g.

Finally note that in steady-state the stock of government debt and the stock of public capital can be positively or negatively associated. This can be seen from the government budget constraint (18), or (17) using (13b)'. Setting time-derivatives to zero:

$$0 = rb + \delta(1 + \frac{\psi\delta}{2})G + c_g - \tau - F_G G$$

which states that if $F_G > \delta(1 + \frac{\psi\delta}{2})$ long run G and b will be positively associated. The interpretation is that the marginal product of government capital exceeds the steady-state expenditures needed to maintain a unit of a constant stock of public capital. In steady-state the depreciation component acts much like a government expenditure item, like c_g, and has a negative association with the long run stock of government debt. Government revenues (such as government earnings from public capital services) have a positive association with the long run stock of b. Thus if $F_G > \delta(1 + \frac{\psi\delta}{2})$, the revenue effect dominates in the long run.

Global Planner

Since the user and rental schemes can be equivalent, I discuss only the rental approach under global planning.

Assume constant returns across all inputs within each country (ie. $F = F_K K + F_L L + F_G G + F_{G*} G^*$ and $F^* = F^*_{K*} K^* + F^*_{L*} L^* + F^*_{G*} G^* + F^*_G G$). The following results depend on this assumption. Using (15b,d) and $-\frac{\dot{\mu}}{\mu} = r - \gamma'$ gives:

(21a) $$q_g(t)G(t) = \int_t^{\infty} e^{-\int_t^s r(u)du}[(F_G + p(s)F^*_G)G(s) - J_g(s)]ds$$

(21b) $$q^*_g(t)G^*(t) = \int_t^{\infty} e^{-\int_t^s r^*(u)du}[(\frac{F^*_G}{p(s)} + F^*_{G^*})G^*(s) - J^*_g(s)]ds$$

Domestic and foreign firms maximize respectively,

(21c) $$V(t) = \int_t^{\infty} e^{-\int_t^s r(u)du}[F(.) - w(s) - J_p(s) - J_g(s)]ds$$

(21d) $$V^*(t) = \int_t^{\infty} e^{-\int_t^s r^*(u)du}[F^*(.) - w^*(s) - J^*_p(s) - J^*_g(s)]ds$$

Substituting (21a,b) into (21c,d) and using (15c,e) yields:

(22a) $$q_g(t)G(t) + q_p(t)K(t) = V(t) - \Omega_{G^*} + \Omega_G$$

where $\Omega_{G^*} = \int_t^{\infty} e^{-\int_t^s r(u)du}[F_{G^*}G^*(s)]ds$ and $\Omega_G = \int_t^{\infty} e^{-\int_t^s r(u)du}[p(s)F^*_G G(s)]ds$

(22b) $$q^*_g(t)G^*(t) + q^*_p(t)K^*(t) = V^*(t) - \Omega^*_G + \Omega^*_{G^*}$$

where $\Omega^*_G = \int_t^{\infty} e^{-\int_t^s r^*(u)du}[F^*_G G(s)]ds$ and $\Omega^*_{G^*} = \int_t^{\infty} e^{-\int_t^s r^*(u)du}[\frac{F_{G^*}}{p(s)}G^*(s)]ds$

Ω_{G^*}, Ω^*_G are the present discounted stream of payments made from one country to the other for the services of (or positive externality generated by) the other's stock of public capital, and Ω_G, $\Omega^*_{G^*}$ represent the corresponding receipts. In other words the global planner sets up a system of international side-payments to create a market for the value of external effects generated by G, G*.

Note that when p(t) is constant[1] so that r(t) = r*(t),

$$V + pV^* = q_pK + q_gG + p\,[q^*{}_pK^* + q^*{}_gG^*]$$

that is, total ownership claims to all the world's stocks of productive capital equal the total world value of firms.

III. Small Open-Economy Model

This section examines the impacts of public investment and public consumption for a small-open economy. Certain properties of this version should aid in understanding the full two-country general-equilibrium model. In this simpler version, there are no distortionary taxes, public capital is provided "free", and σ=1. The interest rate and terms of trade are treated as given. Time subscripts are now omitted except where ambiguities may arise. I first consider an 'exogenous' government and then turn to a national planner.

---

[1] A global planner who weights the welfare of the two countries equally or symmetrically may desire a constant terms of trade, for otherwise one country's purchasing power would rise at the expense of the other's. Such a planner would take actions to offset declining or appreciating terms of trade by judiciously shifting resources between countries. The constancy of p under global planning was not proven formally, but we did show that p equal to 1 or to any other constant was feasible under certain conditions.

Exogenous Government

Substituting the definition of wealth, a, into the equations of motion for aggregate wealth (8d) and consumption (8a), I arrive at the following four state equations to describe the small open-economy:

(23a) $\dot{c} = (r - \rho)c - \lambda\theta\,[z + b + q_p K]$

(23b) $\dot{z} = r\,z + y - c - c_g - J_p - J_g$

(23c) $i_p = \frac{1}{\psi}(q_p - 1)K$

(23d) $\dot{q}_p = (r + \delta)q_p - F_K - \frac{\psi}{2}\left(\frac{i_p}{K}\right)^2$

Linearizing them around steady state gives:

(24)

$$\begin{bmatrix} \dot{c} \\ \dot{z} \\ \dot{K} \\ \dot{q}_p \end{bmatrix} = \begin{bmatrix} (r-\rho) & -\lambda\theta & 0 & -\lambda\theta(K+\frac{q_p r}{F_{KK}}) \\ -1 & r & F_K - \delta(1+\frac{\delta\psi}{2}) & -\frac{q_p K}{\psi} \\ 0 & 0 & 0 & \frac{K}{\psi} \\ 0 & 0 & -F_{KK} & r \end{bmatrix} \begin{bmatrix} dc \\ dz \\ dK \\ dq_p \end{bmatrix}$$

$$
+\begin{bmatrix} -\lambda\theta & 0 & 0 & \lambda\theta\frac{q_pF_{KG}}{F_{KK}} & \lambda\theta\frac{q_pF_{KG^*}}{F_{KK}} \\ 0 & -1 & -(1+\psi\delta) & F_G+\frac{\delta^2\psi}{2} & F_{G^*} \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -F_{KG} & -F_{KG^*} \end{bmatrix} \begin{bmatrix} db \\ dc_g \\ di_g \\ dG \\ dG^* \end{bmatrix}
$$

For each variable x above, let $dx = x - \bar{x}$, where $\bar{x}$ is the steady-state value of x. c, q_p are non-predetermined and K, z are predetermined. The roots are $\kappa_1 = r + \lambda > 0$, $\kappa_2 = r - \theta < 0$,

$$
\kappa_3 = \frac{r + \sqrt{r^2 + \frac{4K}{\psi}(-F_{KK})}}{2} > 0,\ \kappa_4 = \frac{r - \sqrt{r^2 + \frac{4K}{\psi}(-F_{KK})}}{2} < 0.
$$

Thus the above system is saddlepoint stable[1].

The system above is also block-recursive. The dynamics of the q_p, K subsystem feed into the dynamics of the c, z subsystem. Both subsystems will be affected by domestic and foreign public investment shocks. These shocks work their way directly to the c, z subsystem and indirectly via the q_p, K subsystem. Some steady-state impacts of balanced-budget public consumption shocks are:

$$
\left.\frac{dc}{dc_g}\right|_{\Delta c_g = \Delta\tau} = -\frac{\lambda\theta}{\Omega} < 0
$$

[1] κ_4 is negative so long as $F_{KK} < 0$. κ_2 is negative because if there are many economies in the world lending to and borrowing from one another, world equilibrium requires that the steady-state interest rate lie between δ and $\theta = \delta+\lambda$.

$$\left.\frac{dz}{dc_g}\right|_{\Delta c_g = \Delta\tau} = -\frac{(r-\rho)}{\Omega} \quad \text{depends on } \mathrm{sgn}(r-\rho)$$

where $\Omega = (r + \lambda)(\theta - r) > 0$ is a condition needed to ensure that the marginal propensity to consume out of financial wealth is positive.

A long run balanced budget increase in public consumption crowds out long run private consumption[1] - see Figure 2.1. It is assumed that the c_g increase is unanticipated and permanent. Point E_0 denotes an initial equilibrium, E_1 a final equilibrium, and E_2 an intermediate point, if any. The effect on z depends on the sign of $r - \rho$. In steady-state the stock of wealth a ($= b + z + q_pK$) is zero when $r = \rho$, negative when $r < \rho$, and positive when $r > \rho$[2]. When $r < \rho$, the c_g shock lowers c in the long run but by less than the rise in c_g. Thus long run absorption is higher. Across steady states this can only occur if z is higher (that is, a country runs a steady-state trade deficit only if its net stock of foreign assets is positive). To lead to a long run situation in which $z > 0$, home private consumption falls on impact by more than the increase in public consumption; that is, private consumption overshoots its long run equilibrium level. Then, along the transition path to the long run, the economy runs current account surpluses. However when $r > \rho$, long run c falls more than long run c_g

[1] To evaluate the impact when $\lambda = 0$ it is also necessary to set $r=\rho$, a steady state requirement in representative agent models. Then $\theta = \lambda + \rho$. Upon substituting this into Ω, it will be found that public consumption crowds out private consumption one-for-one in the long run.

[2] This can be seen by setting $\dot{c} = \dot{a} = 0$ and solving simultaneously to get:

$$a = \frac{(r - \rho)(F_L - \tau)}{\Omega}.$$

rises; that is, long run absorption is smaller. Thus along the transition path to the long run the economy runs current account deficits. On impact, therefore, c undershoots its long run equilibrium level. When $r = \rho$ public consumption crowds out private consumption one-for-one in the long run, and there are no steady-state or transitional changes in z.

I turn now to long run public capital shocks. Again I assume balanced-budget financing[1] and that shocks are permanent and unanticipated. Some steady-state impacts are:

$$\left.\frac{dc}{dG}\right|_{\Delta b = 0} = \frac{\lambda\theta[\delta(1+\frac{\psi\delta}{2})(\frac{F_{KG}}{-F_{KK}} - 1) + F_G]}{\Omega} > 0 \quad \text{provided } \delta \text{ is not "too" large}$$

$$\left.\frac{dz}{dG}\right|_{\Delta b = 0} = \frac{(r-\rho)[F_G + \frac{F_K F_{KG}}{-F_{KK}} - \delta(1+\psi\delta)] - \lambda\theta q_p \frac{F_{KG}}{-F_{KK}}}{\Omega}$$

In what follows I assume: $[F_G + \frac{F_K F_{KG}}{-F_{KK}} - \delta(1+\psi\delta)] > 0$. That is, a higher steady-state G stock raises output (directly, and indirectly by stimulating an increase in K) sufficiently to keep itself maintained against depreciation. If $F_{KG} < -F_{KK}$, as long as the depreciation rate is not too large, a balanced-budget increase in the long run stock of public capital will increase long run private consumption. The positive effect comes from the impact public capital has on long run output and on private sector wealth through an increase in the stock of K. If steady-state depreciation is

---

1 In this case the required change in long run taxes is $\Delta\tau = \delta(1+\frac{\psi\delta}{2})\Delta G$.

high, it is burdensome to maintain the stock of K and G in the long run because a larger share of resources will be absorbed by depreciation rather than by private consumption. The long run impact on z depends on whether the output effect is stronger or the consumption effect (given by the last term on the numerator of $\frac{dz}{dG}$ above). Whether the output effect will outweigh the latter depends on the sign of (r - ρ). It is necessary but not sufficient that r > ρ in order for long run z to increase - that is, the economy must have a high saving propensity.

The transitional dynamics are depicted in Figure 2.2. Both loci are shifted to the left. When $r \leq \rho$, the economy has a low saving propensity - ie. $a \leq 0$ - and long run z is lower when the long run stock of public capital G is higher. Recall that $a = b + q_pK + z$. K is higher and, by definition, b is constant because of the balanced budget assumption. This leaves z to be negative in order that $a \leq 0$. Because agents desire to hold non-positive wealth and are induced to hold more K in their portfolio, z is crowded out. To reach a situation in which z is lower, the economy must run current account deficits along the transition path; this causes private consumption on impact to overshoot its long run level. When $r > \rho$ there are two possibilities, both associated with a positive long run stock of wealth, a. The first is a repeat of the process just described - long run $a > 0$ even if z falls, provided K increases more. The second possibility is that long run z increases. Agents desire such large positive holdings of wealth that the increase in K is insufficient; z must be higher also. If this is the case, consumption on impact undershoots its long run level and the economy runs current account surpluses along the transition path.

Thus in contrast to balanced-budget unproductive c_g, productive balanced-budget public spending has a tendency to raise long run private consumption for a small open-economy. Unless the economy has a high enough saving propensity, however, in the long run the economy could become a debtor because of the strong stimulus to short run consumption. It should be noted that these results depend on depreciation being not too large, for if the steady-state maintenance expenses were so large as to absorb greater steady-state resources, long run private consumption could be lower. This would be depicted by the $\dot{c} = 0$ locus shifting very far to the left. In this case the private K increase would be such that its contribution to output would fall short of the increase in the cost of maintaining K in steady-state[1].

National Planner

In this economy G is endogenous and a price q_g associated with it is introduced. The model is still block recursive - that is, the dynamics of the (q_g, q_p, G, K) subsystem still feed into the (c,z) subsystem. The new subsystem is:

$$\text{(25a)}\ \dot{q}_p = (r + \delta)q_p - F_K - \frac{\psi}{2}\left(\frac{i_p}{K}\right)^2$$

$$\text{(25b)}\ \dot{q}_g = (r + \delta)q_g - F_G - \frac{\psi}{2}\left(\frac{i_g}{G}\right)^2$$

[1] I have ruled out that this is the case with a higher stock of G by the condition:

$$[F_G + \frac{F_K F_{KG}}{-F_{KK}} - \delta(1+ \psi\delta)] > 0$$

(25c) $\dot{K} = \frac{K}{\psi}(q_p - 1) - \delta K$

(25d) $\dot{G} = \frac{G}{\psi}(q_g - 1) - \delta G$

The linearized model (around steady-state) is:

(26)

$$\begin{bmatrix} \dot{q}_p \\ \dot{q}_g \\ \dot{K} \\ \dot{G} \end{bmatrix} = \begin{bmatrix} r & 0 & -F_{KK} & -F_{KG} \\ 0 & r & -F_{GK} & -F_{GG} \\ \frac{K}{\psi} & 0 & 0 & 0 \\ 0 & \frac{G}{\psi} & 0 & 0 \end{bmatrix} \begin{bmatrix} dq_p \\ dq_g \\ dK \\ dG \end{bmatrix} + \begin{bmatrix} -F_{KG*} \\ -F_{GG*} \\ 0 \\ 0 \end{bmatrix} dG^*$$

Again for each variable x above, $dx = x - \bar{x}$, where $\bar{x}$ is the steady-state value of x. The foreign stock of public capital G* is treated as given. K, G are predetermined and their prices are non-predetermined. The roots of the above subsystem are:

$$\kappa_1 = \frac{r + \sqrt{r^2 - 2e_1 - 2e_2}}{2} > 0, \quad \kappa_2 = \frac{r + \sqrt{r^2 - 2e_1 + 2e_2}}{2} > 0$$

$$\kappa_3 = \frac{r + \sqrt{r^2 - 2e_1 - 2e_2}}{2} < 0, \quad \kappa_4 = \frac{r + \sqrt{r^2 - 2e_1 + 2e_2}}{2} < 0$$

where

$$e_1 = \left(\frac{G}{\psi}F_{GG} + \frac{K}{\psi}F_{KK}\right) < 0, \quad e_2 = \sqrt{\left(\frac{G}{\psi}F_{GG} - \frac{K}{\psi}F_{KK}\right)^2 + \frac{4KG}{\psi^2}F_{KG}^2} > 0 .$$

Saddlepoint stability (two positive and two negative roots) will be ensured if $\sqrt{r^2 - 2e_1 \pm 2e_2} > r > 0$ which holds if $e_1 + e_2 < 0$, which in turn holds if $F_{GG}F_{KK} - F_{KG}^2 > 0$; hence all that is required for saddlepoint stability is the requirement that the production function be concave (locally)[1].

Within this subsystem the time paths of K and G are correlated in response to exogenous shocks. Figure 2.3 depicts the response to a permanent increase in the long run stock of foreign public capital. This is very much like an exogenous productivity shock. If the shock is unanticipated, the transitional path is given by E_0-E_2-E_1; if anticipated, the path is E_0-E_{01}-E_{02}-E_1. Thus locally, if public capital is optimally chosen, we should see the prices of public and private capital moving in concert - ie. their rates of change covarying positively. The valuation of public capital using private market prices may not be a bad approximation in the neighborhood of steady-state, although the subsequent price movements will depend on the size of the shifts in the $\dot{q}_p = 0$ and $\dot{q}_g = 0$ loci, which in turn will hinge on how close the marginal products F_K and F_G are to each other on impact. In initial steady-state, F_K and F_G are equal.

V. Two-Country Model

This section investigates the effects of public spending and spillovers in a full general equilibrium open-economy setting. The two-country

[1] The extension of the second-derivative test to functions of three or more variables is just the condition that the principal minors alternate in sign, beginning with negative own-second partials.

setting appears to be a useful vehicle for highlighting the role played by cross-country technological interdependence[1]. The consequences and repercussions for domestic and foreign national consumption, capital accumulation, current account, and the terms of trade are simultaneously derived in this setting. Table 1 summarizes the two-country model. This section contains two parts: (A) Steady-state properties of the model; (B) Transitional dynamics of the model.

(A) Steady-State

In the long run all state variables are stationary and exogenous variables constant. The long run growth rate is zero. The key long-run equilibrium conditions for the case of exogenous government are, assuming no user fee or rental charges:

(27i) $r = r^*$

(27ii) $q_p = 1 + \psi\delta$ (27iii) $q_p^* = 1 + \psi^*\delta^*$

(27iv) $J_g = \delta(1+\frac{\psi\delta}{2})G$ (27v) $J_g^* = \delta^*(1+\frac{\psi^*\delta^*}{2})G^*$

(27vi) $J_p = \delta(1+\frac{\psi\delta}{2})K$ (27vii) $J_p^* = \delta^*(1+\frac{\psi^*\delta^*}{2})K^*$

---

1 It would be a useful future exercise to increase the number of countries. This would allow for various kinds of asymmetric cases, such as one where a third country creates no spillovers but enjoys "spillins", or where a single-country provides all the spillovers, such as defense protection and R&D investment, but receives no spillins.

(27viii) $(1-\tau_k)F_K=rq_p + \delta(1+ \frac{\psi\delta}{2})$ (27ix) $(1-\tau_k^*)F_K^*=rq_p^*+\delta^*(1+\frac{\psi^*\delta^*}{2})$

(27x) $(1 + \tau_c)c = \theta[a + h]$ (27xi) $(1 + \tau_c^*)c^* = \theta^*[a^* + h^*]$

(27xii) $(1 + \tau_c)(r-\rho)c = \lambda\theta a$ (27xiii) $(1 + \tau_c^*)(r-\rho^*)c^* = \lambda^*\theta^*a^*$

(27xiv) $h = \frac{F_L - \tau}{r + \lambda}$ (27xv) $h^* = \frac{F_L^* - \tau^*}{r + \lambda^*}$

(27xvi) $\theta = r + \lambda - \frac{r-\rho}{\sigma}$ (27xvii) $\theta^* = r + \lambda^* - \frac{r-\rho^*}{\sigma^*}$

(27xviii) $\tau = rb + J_g + c_g - \tau_c\, c - \tau_k\, F_K K$

(27ixx) $\tau^* = rb^* + J_g^* + c_g^* - \tau_c^* c^* - \tau_k^* F^*_K{}^* K^*$

(27xx) $rz = (1-\alpha)c - \alpha^* pc^* = - rz^* p$

(27xxi) $y = \alpha c + \alpha^* pc^* + c_g + J_g + J_p$

(27xxii) $y^* = (1-\alpha)\frac{c}{p} + (1-\alpha^*)c^* + c_g^* + J_p^* + J_g^*$

The steady-state properties of the model will be analyzed from three angles. First, I will look at the comparative steady-state impacts of exogenous variables on <u>international prices</u> - namely the world interest rate r and terms of trade p. I then turn to the impacts on production y and y*, and finally to consumption c and c*.

- International Prices -

For simplicity assume $\sigma = \sigma^* = 1$. Using (27x,xi,xii,xiii):

$$\text{(28a)}\quad c = \frac{-\lambda\theta(F_L - \tau)}{(r(r-\rho)-\lambda\theta)(1 + \tau_c)}\,, \qquad \text{(28b)}\quad c^* = \frac{-\lambda^*\theta^*(F^*_{L^*} - \tau^*)}{(r(r-\rho^*)-\lambda^*\theta^*)(1 + \tau_c^*)}$$

Substituting these into (27xxi,xxii) and totally differentiating gives:

(29)

$$\begin{bmatrix} x_{01} & x_{10} \\ x_{02} & x_{20} \end{bmatrix}\begin{bmatrix} dp \\ dr \end{bmatrix} = \begin{bmatrix} x_{11} & x_{12} & x_{13} & x_{14} & x_{15} & x_{16} & x_{17} & x_{18} & x_{19} & 0 \\ x_{21} & x_{22} & x_{23} & x_{24} & x_{25} & x_{26} & x_{27} & x_{28} & 0 & x_{29} \end{bmatrix}\begin{bmatrix} dG \\ dG^* \\ d\tau \\ d\tau^* \\ d\tau_c \\ d\tau_c^* \\ d\tau_K \\ d\tau_K^* \\ dc_g \\ dc_g^* \end{bmatrix}$$

where

$x_{01} = \alpha^* c > 0$

$$x_{10} = \alpha j_3(2r-\rho)(1+\tau_c) + \alpha^* p j_4(2r-\rho^*)(1+\tau_c^*) + \frac{q_p(F_K - \alpha j_1 F_{LK} - \delta(1+\frac{\psi\delta}{2}))}{-F_{KK}(1-\tau_K)} - \frac{q_p \alpha^* p j_2 F^*_{L^*K^*}}{-F^*_{K^*K^*}(1-\tau_K^*)}$$

$$x_{02} = \frac{-(1-\alpha)c}{p^2} < 0$$

$$x_{20} = (1-\alpha^*) j_4(2r-\rho^*)(1+\tau_c^*) + \frac{(1-\alpha)}{p} j_3(2r-\rho)(1+\tau_c) + \frac{q_p^*(F^*_{K^*} - (1-\alpha^*) j_2 F^*_{L^*K^*} - \delta^*(1+\frac{\psi^*\delta^*}{2}))}{-F^*_{K^*K^*}(1-\tau_K^*)} - \frac{q_p^*(1-\alpha) j_1 F_{LK}}{-pF_{KK}(1-\tau_K)}$$

$$x_{11} = F_G - \alpha j_1 F_{LG} - \alpha^* p j_2 F^*_{L^*G} - \delta(1+\frac{\psi\delta}{2}) + \frac{F_{KG}}{-F_{KK}}(F_K - \alpha j_1 F_{LK} - \delta(1+\frac{\psi\delta}{2})) - \frac{F^*_{K^*G}}{-F^*_{K^*K^*}} \alpha^* p j_2 F^*_{L^*K^*}$$

$$x_{12} = F_{G^*} - \alpha j_1 F_{LG^*} - \alpha^* p j_2 F^*_{L^*G^*} + \frac{F_{KG^*}}{-F_{KK}}(F_K - \alpha j_1 F_{LK} - \delta(1+\frac{\psi\delta}{2})) - \frac{F^*_{K^*G^*}}{-F^*_{K^*K^*}} \alpha^* p j_2 F^*_{L^*K^*}$$

$$x_{13} = \alpha j_1 > 0$$

$$x_{14} = \alpha^* p j_2 > 0$$

$$x_{15} = \alpha j_3 \, (r(r-\rho) - \lambda\theta) > 0$$

$$x_{16} = \alpha^* p j_4 (r(r-\rho^*) - \lambda^*\theta^*) > 0$$

$$x_{17} = \frac{-F_K}{-F_{KK}(1-\tau_K)}(F_K - \alpha j_1 F_{LK} - \delta(1+\frac{\psi\delta}{2}))$$

$$x_{18} = \frac{F^*_{K^*}}{-F^*_{K^*K^*}(1-\tau_K^*)}\alpha^* p j_2 F^*_{L^*K^*} > 0$$

$$x_{19} = -1 < 0$$

$$x_{21} = F^*_G - \frac{(1-\alpha)}{p} j_1 F_{LG} - (1-\alpha^*)j_2 F^*_{L^*G} + \frac{F^*_{K^*G}}{-F^*_{K^*K^*}}(F^*_{K^*} - (1-\alpha^*)j_2 F^*_{L^*K^*} - \delta^*(1+\frac{\psi^*\delta^*}{2})) - \frac{F_{KG}}{-F_{KK}}\frac{(1-\alpha)}{p} j_1 F_{LK}$$

$$x_{22} = F^*_{G^*} - \frac{(1-\alpha)}{p} j_1 F_{LG^*} - (1-\alpha^*)j_2 F^*_{L^*G^*} - \delta^*(1+\frac{\psi^*\delta^*}{2}) + \frac{F^*_{K^*G^*}}{-F^*_{K^*K^*}}(F^*_{K^*} - (1-\alpha^*)j_2 F^*_{L^*K^*} - \delta^*(1+\frac{\psi^*\delta^*}{2})) - \frac{F_{KG^*}}{-F_{KK}}\frac{(1-\alpha)}{p} j_1 F_{LK}$$

$$x_{23} = \frac{(1-\alpha)}{p} j_1 > 0$$

$$x_{24} = (1-\alpha^*) j_2 > 0$$

$$x_{25} = \frac{(1-\alpha)}{p} j_3 \lambda\theta > 0$$

$$x_{26} = (1-\alpha^*) j_4 \lambda^*\theta^* > 0$$

$$x_{27} = \frac{F_K}{-F_{KK}(1-\tau_K)}\frac{(1-\alpha)}{p} j_1 F_{LK} > 0$$

$$x_{28} = \frac{-F^*_{K^*}}{-F^*_{K^*K^*}(1-\tau_K^*)}(F^*_{K^*} - (1-\alpha^*)j_2 F^*_{L^*K^*} - \delta^*(1+\frac{\psi^*\delta^*}{2}))$$

$$x_{29} = -1 < 0$$

where $\Delta = x_{01} x_{20} - x_{10} x_{02}$, and

$$j_1 = \frac{-\lambda\theta}{(r(r-\rho) - \lambda\theta)(1+\tau_c)} > 0, \qquad j_2 = \frac{-\lambda^*\theta^*}{(r(r-\rho^*) - \lambda^*\theta^*)(1+\tau_c^*)}$$

> 0,

$$j_3 = \frac{(F_L-\tau)j_1^2}{\lambda\theta} > 0, \qquad j_4 = \frac{(F_L^*-\tau^*)j_2^2}{\lambda^*\theta^*} > 0,$$

under the assumptions that $\lambda\theta > r(r - \rho)$ and $\lambda^*\theta^* > r(r - \rho^*)$. These conditions ensure that the marginal propensity to consume out of financial wealth is positive - see (27xxii,xxiii). From (28a,b) it can be seen that j1, j2 are the marginal propensities to consume out of labour income less taxes[1].

Whether Δ is positive or negative depends on whether x_{10}, x_{20} are both positive or both negative. They will both be positive if a higher world interest rate results in an excess demand for both the domestic and foreign good. A higher world interest rate lowers the steady-state stocks of capital and output, lowers steady-state wages, but raises steady-state asset income; the assumption here is that on net there is excess demand.

With $\Delta > 0$, the following steady-state impacts can be derived (evaluated at $\tau_c = \tau_c^* = \tau_K = \tau_K^* = 0$):

$$\text{sgn}\ \frac{dr}{d\tau} = \text{sgn}\ (x_{01}x_{23} - x_{13}x_{02}) > 0$$

[1] Definitions j_3, j_4 emerge when differentiating consumption with respect to the consumption tax rate and interest rate.

$$\text{sgn}\ \frac{dp}{d\tau} = \text{sgn}\ (x_{13}x_{20} - x_{23}x_{10})\ ?$$

Figure 2.4a depicts the impact of an increase in the home country's steady-state lump-sum taxes[1]. yy is the goods market equilibrium locus for the home good and is downward-sloping because as r rises an excess demand for home output is assumed to result, requiring the relative price of the foreign good, p, to fall (or the relative price of the home good to rise). Similarly y*y* is the goods market equilibrium locus for the foreign good and is upward-sloping because a higher r requires a higher p (or higher relative price for the foreign good) to eliminate an incipient excess demand for the foreign good. An increase in τ shifts both loci upward (since x_{23}, $x_{13} > 0$). The initial equilibrium is given by E_0 and final by E_1. A higher steady-state level of τ is associated with a higher stock of government debt, b, which is net wealth to domestic consumers. Domestic consumers thus increase their steady-state consumption of both goods. The net effect on the terms of trade of an increase in the consumption of both goods is ambiguous.

The steady-state impacts of other taxes are:

$$\text{sgn}\ \frac{dr}{d\tau_k} = \text{sgn}\ (x_{01}x_{27} - x_{17}x_{02})\ ?$$

$$\text{sgn}\ \frac{dp}{d\tau_k} = \text{sgn}\ (x_{17}x_{20} - x_{27}x_{10}) < 0$$

---

[1] Recall that there are two components to lump-sum taxes: an exogenous component and an endogenous component. The steady-state derivatives are taken with respect to the exogenous component.

$$\text{sgn}\ \frac{dr}{d\tau_c} = \text{sgn}\ (x_{01}x_{25} - x_{15}x_{02}) > 0$$

$$\text{sgn}\ \frac{dp}{d\tau_c} = \text{sgn}\ (x_{15}x_{20} - x_{25}x_{10})\ \ ?$$

The impact of higher consumption taxes mimics that of higher lump-sum taxes; that is, a larger stock of government debt is associated with higher consumption tax revenue in the long run. If x_{17} is sufficiently negative it is possible that the long run interest rate can fall in response to a higher capital income tax rate. $x_{17} < 0$ requires $F_K - \alpha j_1 F_{LK} - \delta(1 + \frac{\psi\delta}{2}) > 0$. That is, a rise in τ_k must lower supply (by lowering the stock of capital K - the first term) by more than it lowers domestic absorption (ie. the sum of the second term - which represents the change in consumption due to a change in labour income - and the third term - which represents steady-state depreciation per unit of capital K). The steady-state interest rate must then fall to eliminate an incipient excess demand. This case is depicted in Figure 2.4b.

I turn now to some expenditure impacts. I assume that increases in long run spending are all deficit-financed:

$$\text{sgn}\ \frac{dr}{dc_g} = \text{sgn}\ (-x_{19}\ x_{02}) < 0$$

$$\text{sgn}\ \frac{dp}{dc_g} = \text{sgn}\ (x_{19}\ x_{20}) < 0$$

$$\text{sgn}\ \frac{dr}{dG} = \text{sgn}\ (x_{01}\ x_{21} - x_{11}\ x_{02}) < 0$$

$$\text{sgn} \frac{dp}{dG} = \text{sgn} (x_{20}\, x_{11} - x_{21}\, x_{10}) \ ?$$

The impact of c_g is shown in Figure 2.4c. Higher public consumption is associated with a lower long run interest rate. This results because across steady-states, higher c_g is associated with a lower stock of public debt. An increased demand for domestic goods by the government appreciates the terms of trade. In Buiter (1987) the impact of c_g on p is ambiguous. The difference in result is due to the fact that the domestic government in his model purchases both domestic and foreign goods; in this model, c_g falls exclusively on domestic output.

The impact of a higher long run G depends on the signs of x_{21}, x_{11}. If these are sufficiently negative, long run global crowding in can occur - as depicted in Figure 2.4d. Their signs are negative if a higher stock of public capital and higher induced stocks of private capital increase long run production but increase steady-state absorption even more so as to produce an incipient excess demand. The steady-state world interest rate must then fall. From x_{21}, x_{11}, the following conditions help improve this possibility: (i) large cross-country spillover effects - particularly on raising the wages of the other country - along with significant enough import propensities (α^*, $1-\alpha$); (ii) significant enough depreciation rates so that in steady-state a significant enough share of resources is absorbed into maintaining the steady stocks of capital - but not too much as to crowd out steady-state consumption[1]; (iii) high labour shares of output (F_L, $F^*_{L^*}$) which help

[1] For normative reasons only. As a positive feature, higher steady-state depreciation contributes to creating incipient excess demand.

magnify the impact of increased public capital on human wealth; (iv) large j_1, j_2, ie. the marginal propensities to consume out of after-tax labour income.

In the steady-state analyses so far, the impacts on production and consumption are implicit, the focus being on the effects on world market clearing prices. In the next couple of subsections I isolate the impacts on production and consumption, conditional on world prices not having changed. To derive the total effect on production or consumption, one must therefore evaluate the direct and indirect effects of a change in some exogenous variable X as:

$$\frac{\Delta y}{\Delta X} = \left.\frac{\partial y}{\partial X}\right|_{\text{direct}} + \left.\frac{\partial y}{\partial r}\frac{\partial r}{\partial X}\right|_{\text{indirect}} + \left.\frac{\partial y}{\partial p}\frac{\partial p}{\partial X}\right|_{\text{indirect}}$$

$$\frac{\Delta c}{\Delta X} = \left.\frac{\partial c}{\partial X}\right|_{\text{direct}} + \left.\frac{\partial c}{\partial r}\frac{\partial r}{\partial X}\right|_{\text{indirect}} + \left.\frac{\partial c}{\partial p}\frac{\partial p}{\partial X}\right|_{\text{indirect}}$$

The next two subsections will study some of these "direct" impacts.

- Production -

Recall that when $\dot{q}_p=0$, $(1-\tau_k)F_K = rq_p + \delta(1 + \frac{\psi\delta}{2})$. Totally differentiating this and substituting the result into the totally differentiated production function $dY = F_K dK + F_G dG + F_{G*}dG^*$ (assuming $dL=0$ since labour is fixed in supply), I obtain the steady-state output effects of public capital, the capital income tax rate, and the world interest rate (again evaluated at $\tau_k = \tau_k^* = \tau_c = \tau_c^* = 0$)[1, 2]:

$$(30)\quad dY = \beta_G dG + \beta_{G^*} dG^* + \beta_{\tau_K} d\tau_K + \beta_r\, dr$$

where $\beta_G = F_G - \frac{F_K}{F_{KK}} F_{KG} > 0,\qquad \beta_{G^*} = F_{G^*} - \frac{F_K}{F_{KK}} F_{KG^*} > 0$

$\beta_{\tau_K} = \frac{F_K^2}{F_{KK}} < 0,\qquad \beta_r = q_p \frac{F_K}{F_{KK}} < 0$

Alternatively under national planning, G is endogenous. From $\dot{q}_g=0$, $F_G = rq_g + \delta(1 + \frac{\psi\delta}{2})$. Totally differentiating this and substituting the result into (30), we get:

$$(31)\quad dY = \beta_{G^*} dG^* + \beta_{\tau_K} d\tau_K + \beta_r\, dr$$

where

$$\beta_{G^*} = F_{G^*} + \frac{F_K}{F_{KK}F_{GG} - F_{KG}^2}\left[F_{GG^*}F_{KG} + F_{KG^*}F_{KG} - F_{KK}F_{GG^*} - F_{GG}F_{KG^*}\right] > 0$$

---

1 It is assumed that all cross partials - eg. F_{KG} , ... - are positive (ie. all inputs are complements) and that principal minors alternate in sign beginning with negative own second partials - eg. $F_{KK} < 0$, $F_{GG} < 0$, and so forth.

2 This type of production function is estimated - in both levels and first differences - in my empirical work with the exception that labour hours is not constrained to be fixed. I find β_G, $\beta_{G^*} > 0$ (corrected for capacity utilization) but tend to find β_{τ_k} to be insignificant and sometimes positive. The latter finding could indicate the presence of a public policy feedback rule relating capital income tax rates to changes in output growth and/or cycles. It would therefore be desirable to derive and analyze decision rules for taxes also.

$$\beta_{\tau_K} = \frac{-F_K^2(F_{GK}-F_{GG})}{F_{KK}F_{GG} - F_{KG}^2} < 0, \qquad \beta_r = q_p \frac{F_K(F_{KK}+F_{GG}-2F_{KG})}{F_{KK}F_{GG} - F_{KG}^2} < 0$$

Under global planning, all capital stocks G, G*, K , K* are endogenous. Setting $\dot{q}_g=0$, $\dot{q}_g{}^*=0$, $\dot{q}_p=0$, $\dot{q}_p{}^*=0$, we obtain the following steady-state marginal productivity conditions: $F_K = F_G + pF^*_G = F^*_{G^*} + \frac{F_{G^*}}{p} = F^*_{K^*}$. After totally differentiating them and substituting the results into the global production function, we arrive at:

(32) $d(Y + pY^*) = \beta_p\, dp + \beta_r\, dr + \beta_{\tau_K} d\tau_K + \beta_{\tau_{K^*}} d\tau_{K^*}$.

The coefficient details are omitted. I find that as long as the production functions F and F* are each (locally) concave, $\beta_r < 0$, $\beta_{\tau_K} < 0$, and $\beta_{\tau_{K^*}} < 0$. The value of β_p is $\lesseqgtr 0$ according to whether:

$$\frac{pF^*_G}{-(F_{GG} + pF^*_{GG})} \lessgtr \frac{F_{G^*}}{-(F_{G^*G^*} + pF^*_{G^*G^*})}$$

A rise in steady-state p has ambiguous effects on the steady state value of global output (Y + pY*). The supplies of Y* and Y may change in response to the higher relative price of the foreign good. In particular if p is higher, the restoration of steady-state marginal productivity conditions (ie. $\dot{q}_g = \dot{q}_p = \dot{q}_p{}^* = \dot{q}_g{}^*=0$) requires that G rise and G* fall[1], the

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[1] This requires that the sum of pairs of own second partials (ie. $F_{KK}\,F_{GG}$ + $F_{GG}\,F_{G^*G^*}$ + $F_{KK}\,F_{G^*G^*}$ +....) be <u>greater in absolute value</u> than the sum of pairs of cross partials (ie. $F_{KG}\,F_{KG^*}$ + $F_{G^*K}F_{G^*G}$ + $F_{GG^*}F_{GK}$ +....) . The reason is that when p is higher, to maintain $\dot{q}_g=0=\dot{q}_p$, both G and G* must rise (so as to reduce F_G and F^*_G); on the other hand to maintain $\dot{q}_g{}^*=0=$

intuition being that when p is higher, the global planner finds it cheaper to use domestic output in production. The above expression states that whether the value of global output rises hinges on the positive output effect of a higher G on the production of Y* being greater than the adverse effect of a lower G* on the production of Y. On the other hand if p falls, the value of global output rises if the increase in G* raises Y more than the decrease in G reduces Y*. If production functions are identical and symmetric (G* enters F(.) in the same way G enters F*(.)), β_p is zero.

In both the national and global planner cases, the consumption tax rate does not appear in the reduced-form production functions. This is not to say that consumption taxes have no effect on supply; their effects enter by influencing the world interest rate or terms of trade.

- Consumption -

In this subsection I limit the analysis to exogenous governments only (the main changes to be made when considering endogenous governments are to incorporate the steady-state marginal productivity conditions discussed above). In order to analyze steady-state consumption and external asset/debt holdings, I work with the following three equations :

$\dot{q}_p^*$, both G and G* must fall (so as to raise F_{G*} and F^*_{G*}). The restriction just given is that in the first set of conditions $\dot{q}_g=0=\dot{q}_p$, a G increase dominates a G* decrease, while in the second set of conditions $\dot{q}_g^*=0=\dot{q}_p^*$, a G* decrease dominates a G increase.

(33a) $(1 + \tau_c)c = \theta[a + h]$

(33b) $(1 + \tau_{c*})c^* = \theta^*[a^* + h^*]$

(33c) $rz = (1-\alpha)c - \alpha^* pc^*$

Equations (33a, b) are the home and foreign consumption functions. The LHS of (33c) is the steady-state home country service account balance and the RHS the steady-state trade balance. Totally differentiating (33a-33c), evaluated at zero distortionary taxes, I arrive at:

(34)

$$\begin{bmatrix} c_{11} & 0 & c_{12} \\ 0 & c_{21} & c_{22} \\ -z_2 & -z_3 & 1 \end{bmatrix} \begin{bmatrix} dc \\ dc^* \\ dz \end{bmatrix} =$$

$$\begin{bmatrix} c_{13} & c_{14} & c_{15} & 0 & c_{16} & 0 & c_{17} & 0 & c_{18} & 0 & c_{19} & 0 \\ c_{24} & c_{23} & 0 & c_{25} & c_{26} & c_{30} & 0 & c_{27} & 0 & c_{28} & 0 & c_{29} \\ 0 & 0 & 0 & 0 & z_1 & z_4 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} dG \\ dG^* \\ dc_g \\ dc_g^* \\ dr \\ dp \\ d\tau \\ d\tau^* \\ d\tau_K \\ d\tau_K^* \\ d\tau_c \\ d\tau_c^* \end{bmatrix}$$

where

$$c_{11} = \frac{1}{\theta} > 0, \qquad c_{12} = -1 < 0,$$

$$c_{13} = q_p \frac{F_{KG}}{-F_{KK}} - \frac{\delta(1+\psi\delta)}{r} + \frac{1}{r+\lambda}[F_{LG} + \frac{F_{LK}F_{KG}}{-F_{KK}}]$$

$$c_{14} = q_p \frac{F_{KG^*}}{-F_{KK}} + \frac{1}{r+\lambda}[F_{LG^*} + \frac{F_{LK}F_{KG^*}}{-F_{KK}}] > 0$$

$$c_{15} = -\frac{1}{r} < 0$$

$$c_{16} = \frac{-q_p^2}{-F_{KK}} - \frac{b}{r} - \frac{1}{r+\lambda}[h + \frac{F_{LK}q_p}{-F_{KK}}] < 0$$

$$c_{17} = \frac{1}{r} - \frac{1}{r+\lambda} > 0$$

$$c_{18} = \frac{-q_pF_K}{-F_{KK}} + \frac{F_KK}{r} - \frac{1}{r+\lambda}[\frac{F_{LK}F_K}{-F_{KK}}]$$

$$c_{19} = \frac{c}{r} - \frac{c}{\theta} > 0$$

$$c_{21} = \frac{1}{\theta^*} > 0, \qquad c_{22} = \frac{1}{p} > 0,$$

$$c_{23} = q_p^* \frac{F^*_{K^*G^*}}{-F^*_{K^*K^*}} - \frac{\delta^*(1+\psi^*\delta^*)}{r} + \frac{1}{r+\lambda^*}[F^*_{L^*G^*} + \frac{F^*_{L^*K^*}F^*_{K^*G^*}}{-F^*_{K^*K^*}}]$$

$$c_{24} = q_p^* \frac{F^*_{K^*G}}{-F^*_{K^*K^*}} + \frac{1}{r+\lambda^*}[F^*_{L^*G} + \frac{F^*_{L^*K^*}F^*_{K^*G}}{-F^*_{K^*K^*}}] > 0$$

$$c_{25} = -\frac{1}{r} < 0$$

$$c_{26} = \frac{-q_p{}^{*2}}{-F^*{}_{K^*K^*}} - \frac{b^*}{r} - \frac{1}{r+\lambda^*}[h^* + \frac{F^*{}_{L^*K^*}q_p{}^*}{-F^*{}_{K^*K^*}}] < 0$$

$$c_{27} = \frac{1}{r} - \frac{1}{r+\lambda^*} > 0$$

$$c_{28} = \frac{-q_p{}^*F^*{}_{K^*}}{-F^*{}_{K^*K^*}} + \frac{F^*{}_{K^*}K^*}{r} - \frac{1}{r+\lambda^*}[\frac{F^*{}_{L^*K^*}F^*{}_{K^*}}{-F^*{}_{K^*K^*}}]$$

$$c_{29} = \frac{c^*}{r} - \frac{c^*}{\theta^*} > 0, \qquad c_{30} = \frac{z}{p^2}$$

$$z_1 = -\frac{z}{r}, \; z_2 = \frac{(1-\alpha)}{r} > 0, \; z_3 = -\frac{\alpha^*p}{r} < 0, \; z_4 = -\frac{\alpha^*c^*}{r} < 0.$$

As long as $\lambda > 0$, government debt is viewed as net wealth and c_{17}, c_{27} are positive; that is, in c_{17}, c_{27} the first terms reflect the fact that a higher long run τ is associated with a larger long run stock of debt b, and the second terms indicate that future taxes on human wealth are discounted more heavily. Again I assume θ, θ^* exceed r for stability reasons, and hence c_{19}, c_{29} are positive. As already noted, higher consumption taxes across steady-states are associated with a larger stock of government debt and greater private consumption. On the other hand, the impacts of capital income taxes are ambiguous because while they too are associated with a larger steady-state stock of government debt, they lower the steady-state stock of private capital and reduce private wealth.

If depreciation is small enough, $c_{13}, c_{23} > 0$, so that G and G* are associated with greater long run consumption. Once again the steady-state absorption share of maintaining public capital must not be "too" large. c_{14}, c_{24} are unambiguously positive because they are the spillover terms: agents can enjoy greater consumption allowed by an increase in spillover capital without having to finance it through higher long run taxes or maintain it against depreciation. c_{16}, c_{26} are negative because a higher interest rate works to depress human wealth (via heavier discounting) and non-human wealth by reducing the stock of private capital and by reducing the present value of government bonds.

The determinant of the LHS matrix in (34) (evaluated at $\tau_c = \tau_{c^*}$ 0) is $\lesseqgtr 0$ according to whether:

$$(34)' \quad (1-\alpha)\theta + \alpha^*\theta^* \gtreqless r$$

(34)' has a 'transfer criterion' interpretation: the LHS is the sum of the marginal propensities to import by both countries; the RHS is the amount of wealth transferred between countries per external claim or liability. To see this, recall that in steady-state the capital account balance of the home country is zero:

ie. $rz = (1-\alpha)c - \alpha^*pc^*$.

In steady-state, a net creditor country ($z > 0$) runs a steady-state trade deficit while a net debtor ($z < 0$) runs a steady-state trade surplus. Differentiating the above with respect to z results in:

$$r + z\frac{dr}{dz} = (1-\alpha)\frac{dc}{dz} - \alpha^*p\frac{dc^*}{dz} - \alpha^*c\frac{dp}{dz}$$

or (34)" $$r = (1-\alpha)\theta + \alpha^*\theta^* - z\frac{dr}{dz} - \alpha^* c\frac{dp}{dz}$$

where I made use of the consumption functions $c=\theta[z + q_pK + b + h]$, $c^*=\theta^*[z^* + q_p^*K^* + b^* + h^*]$, and $z^* = -\frac{z}{p}$. From (34)", it can be seen that (34)' reflects the effect of an increase in Δz on world import demands absent changes in world prices (r, p). Thus (34)' will signal the changes needed in r and p.

To illustrate the use of (34)', I abstract from price changes. If $(1-\alpha)\theta + \alpha^*\theta^* > r$, an increase in wealth of Δz for the creditor country will produce a more than proportionate increase in the creditor country's steady-state trade deficit. That is, the creditor country raises its import demand while the debtor country reduces its import demand (or demand for the creditor's exports) in response to a loss in wealth $-\Delta z^*$. The transfer Δz therefore results in a higher demand for the debtor country's output, working to increase the debtor's terms of trade. Similarly, if $(1-\alpha)\theta + \alpha^*\theta^* < r$, a transfer of wealth to the creditor country will produce a less than proportionate expansion in the creditor's trade deficit, resulting in a relatively greater increase in the demand for the creditor country's own good, thus working to increase the creditor's terms of trade.

In summary, the interpretation of (34)' is that if $(1-\alpha)\theta + \alpha^*\theta^* > r$ (labelled case 1), there is an incipient increase in the demand for the output of the 'transferer' country; if $(1-\alpha)\theta + \alpha^*\theta^* < r$ (labelled case 2), there is an incipient increase in the demand for the output of the 'transferee' country.

Returning to the system in (34), I substitute the third equation (for dz) into the other two equations. This yields two consumption equilibrium conditions. Figures 2.5a, 2.5b depict two possible configurations. In each, the vertical axis measures total home consumption expenditures, the horizontal total foreign consumption expenditures. The c_0 locus is downward sloping because an increase in c^* requires, across steady-states, a larger stock of net external claims z^* (or lower z) which reduces domestic nonhuman wealth and thus reduces c. The c_0^* locus is likewise also downward-sloping. Note that it is possible to ascertain the value of z from the diagrams. There is a locus of points c, c^* such that $z = 0$:

ie. $0 = rz = (1-\alpha)c - \alpha^* pc^*$

or $\frac{c}{c^*} = \frac{\alpha^* p}{1-\alpha}$.

This locus is depicted in each figure; it need not of course coincide with the 45^0 line. Moreover the c_0, c_0^* need not have intersected at a point where $z = 0$. I have chosen $z = 0$ for expositional reasons. North of the $z = 0$ locus is the region where $z > 0$ - that is, for each c^*, higher steady-state c is possible only if steady-state z is higher. South of $z = 0$ is the region where $z < 0$.

Figure 2.5a illustrates case 1, where the home country's consumption equilibrium locus, c_0, is steeper than the foreign country's consumption equilibrium locus, c_0^*. Figure 2.5b illustrates case 2, where the slopes are reversed[1,2]. The slope configurations imply that under case 1 there is a

1 The slopes are:

higher demand for the debtor country's output when the creditor country accumulates more net external assets, and that under case 2 there is a higher demand for the creditor country's own output when the creditor country accumulates more net external assets. For example under case 1, when $z > 0$, the home country's steady-state trade deficit along the c_0 locus is larger than the foreign country's steady-state trade surplus along the c_0^* locus. Thus disequilibria in this region are characterized by higher net import demands[1], consistent with the transfer criterion $(1-\alpha)\theta+\alpha^*\theta^* > r$. Moreover, as z increases in this region, the home country's steady-state trade deficit along c_0 and the foreign country's steady-state trade surplus along c_0^* widen. But since the c_0 locus is steeper than the c_0^* locus, the consequent increase in the home country's import demand is more pronounced than the consequent increase in the foreign country's export supply, and there is an incipient excess demand for foreign output. It is in this sense that when net external assets are accumulated by a creditor, a net increase in the demand for the debtor's output results. In contrast, under case 2, when $z > 0$, the situation is reversed: the home country's steady-

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$$\left.\frac{dc}{dc^*}\right|_{c_0\,\text{locus}} = \frac{-\dfrac{\theta\alpha^*p}{r}}{\left[1-\dfrac{\theta(1-\alpha)}{r}\right]}, \quad \left.\frac{dc}{dc^*}\right|_{c_0^*\text{locus}} = \frac{\left[1-\dfrac{\theta^*\alpha^*}{r}\right]}{-\dfrac{\theta^*(1-\alpha)}{rp}}$$

2

I restrict $(1 - \frac{\theta^*\alpha^*}{r})$, $(1 - \frac{\theta(1-\alpha)}{r})$ to be positive. These prevent the marginal propensity to import out of service income from exceeding 100%, as this would not be sustainable in steady state. Moreover, the c_0, c_0^* loci would not be downward sloping, and the marginal propensity to consume from other income would be negative.

[1] ie. the sum of home and foreign import demands is greater than the sum of home and foreign export supplies.

state trade deficit along the c_0 locus is smaller than the foreign country's steady-state trade surplus along the c_0^* locus. Disequilibria in this region involve lower net import demands[1], and as net external assets are accumulated by a creditor, a relatively greater increase in demand for the creditor country's own output results.

I turn now to some comparative steady-state impacts. I examine the impacts of deficit-financed changes in long run public consumption and public investment on long run consumption and external assets, conditional on r and p remaining unchanged. I then turn to the impacts that changes in the world interest rate and terms of trade have on long run consumption and external assets. The initial steady-state z is assumed to be zero.

The impacts of public consumption are in Figures 2.6a and 2.6b. With c_{15} negative, the home consumption equilibrium locus shifts down and to the left: ceterus paribus, an increase in home public consumption takes resources away from private consumption[2]. Under case 1, however, the unilateral increase in home public consumption can 'crowd' in own private consumption, at the expense of foreign private consumption, while under case 2 the same policy can increase foreign private consumption at the expense of home private consumption. The intuition is that the public consumption shock transfers wealth to the foreign economy (as domestic absorption is higher), but under case 1, the transfer significantly raises the foreign economy's import demand for domestic output while the loss of home wealth reduces the home country's demand for foreign relative to

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[1] ie. the sum of home and foreign import demands is less than the sum of home and foreign export supplies.

[2] Note that if public consumption were balanced-budget (lump-sum tax) financed instead, the c_0 locus would still shift to the left on net (ie. the sum of c_{15} and c_{17} is still negative).

domestic output; thus across steady states, the actual equilibrium occurs in the region $z > 0$. The home economy actually ends up being the net creditor and gets to enjoy steady-state trade deficits. Under case 2, the 'transfer' to the foreign economy results instead in an overall net increase in the demand for foreign output; the foreign economy is thus the long run creditor.

Figures 2.6c, 2.6d turn to the impacts of a higher long run stock of domestic public capital. This time regardless of the direction of the transfer criterion, long run private consumption of both countries can rise. The final impact on long run net external assets is ambiguous, though the diagrams have been drawn so that the new equilibria remain on the z=0 locus. It is also possible that if the size of the shifts is asymmetric, one country's long run consumption can increase at the expense of the other's. That both the c_0, c_0* loci shift out is attributable to the presence of spillovers. In the absence of spillovers an increase in long run G leads, under case 1, to an equilibrium point like E_1 in Figure 2.6a. Again because of the transfer effect the increase in home country wealth (from increased production and income) results in an overall increase in demand for foreign output and an equilibrium where the home economy ends up being a steady-state debtor. The spillover effect thus works to boost foreign consumption so that the world equilibrium may occur at a point like E_2.

The long run impacts of a higher world interest rate on home and foreign consumption - evaluated at z = 0 - can be obtained from (34):

$$\text{(35a)} \quad \frac{dc}{dr} = M^{-1} \left[\; c_{16} \left(\frac{1}{\theta^*} - \frac{\alpha^*}{r} \right) - c_{26} \frac{\alpha^* p}{r} \right]$$

(35b) $\frac{dc^*}{dr} = M^{-1}[\, c_{26}(\frac{1}{\theta} - \frac{1-\alpha}{r}) - c_{16}\frac{1-\alpha}{pr}]$

where

$$c_{26} = \frac{\partial(b^* + q_p{}^*K^* + h^*)}{\partial r} < 0\ ,\ c_{16} = \frac{\partial(b + q_pK + h)}{\partial r} < 0$$

and M is the determinant of (34), and sgn M $\gtrless$ 0 as r $\gtrless$ $(1-\alpha)\theta + \alpha^*\theta^*$.

Now, if $c_{26} = c_{16}$, both $\frac{dc}{dr} < 0$, $\frac{dc^*}{dr} < 0$. This can be shown as follows. First, $c_{26} = c_{16}$ if the initial stocks of comprehensive wealth in both countries are equal - ie. $a + h = a^* + h^*$. Secondly, since $z = 0$, there is initially balanced trade: $rz = 0 = (1-\alpha)c - \alpha^*pc^*$. These two points imply that $(1-\alpha)\theta = \alpha^*p\theta^*$. Substituting this into (35a) and (35b) will show that the numerator (excluding c_{26} or c_{16}) has the same sign as the denominator. Since c_{26}, c_{16} are negative, this produces the desired result.

Instead if (i) $|c_{26}| << |c_{16}|$

(a) $\frac{dc}{dr} > 0$, $\frac{dc^*}{dr} < 0$ when $r < (1-\alpha)\theta + \alpha^*\theta^*$

(b) $\frac{dc}{dr} < 0$, $\frac{dc^*}{dr} > 0$ when $r > (1-\alpha)\theta + \alpha^*\theta^*$

and if (ii) $|c_{26}| >> |c_{16}|$

$$\text{(a)}\quad \frac{dc}{dr} < 0,\ \frac{dc^*}{dr} > 0 \quad \text{when } r < (1-\alpha)\theta + \alpha^*\theta^*$$

$$\text{(b)}\quad \frac{dc}{dr} > 0,\ \frac{dc^*}{dr} < 0 \quad \text{when } r > (1-\alpha)\theta + \alpha^*\theta^*$$

All of these possibilities are shown in Figures 2.7a, 2.7b. The final impacts depend on the values of c_{16}, c_{26}. These coefficients represent the effects of a change in the world interest rate on non-human wealth (excluding z and z*) and human wealth in the home and foreign country respectively. Typically if a higher long run world interest rate depresses home and foreign wealth evenly, both countries will be consuming less in the long run, regardless of the direction of the transfer criterion. However if the higher long run world interest rate has an uneven impact on home and foreign wealth, there may be a possibility that one country's consumption rises at the expense of the other's; these are shown by points 1 and 3 in Figures 2.7a, 2.7b. Which country it is that gets to have the higher long run consumption level depends on the transfer criterion and on whose wealth it is that gets depressed more. If it is the home country whose wealth is depressed more $|c_{26}| << |c_{16}|$, the foreign country is relatively wealthier, and under case 1 of the transfer criterion, there will be a relatively greater increase in demand for the home country's output so that in the long run the home country will be a creditor and enjoy higher long run consumption. Instead under case 2 of the transfer criterion, there will be a relatively greater increase in demand for foreign country output, and an opposite long run scenario will emerge.

Finally the long run impact of a change in the terms of trade (evaluated at z=0) is:

$$(36) \quad \frac{dc}{dp} = M^{-1} [-c_{12} z_4 (c_{21} + z_3 c_{22}) + c_{12} c_{22} z_3 z_4]$$

where M is the determinant of (34), and the numerator is negative.

Thus $\frac{dc}{dp} > 0$, if $M < 0$ (ie. $r < (1-\alpha)\theta + \alpha^*\theta^*$) and $\frac{dc}{dp} < 0$, if $M > 0$, as shown in Figures 2.7c, 2.7d. The intuition is that an improvement in the terms of trade for the foreign country (ie. a p increase) makes the foreign country wealthier, and as before, if case 1 holds, will result in a net increase in the demand for home country output and in the home country becoming a creditor in the long run. If case 2 holds, the opposite outcome will result. Note that the z = 0 locus rotates to the left since p is higher.

B. Dynamic Simulations

The simulation experiments presented here are of three types: (a) public spending shocks (namely, public investment (i_g) and public consumption (c_g) shocks); (b) intertemporal redistributions of lump-sum taxes; (c) revenue-neutral distortionary tax reforms.

For (a) I have examined both tax and bond-financed increases in public spending; permanent and temporary; anticipated and unanticipated; unilateral and globally coordinated. For (b) I have considered current tax cuts followed by future tax increases; current tax increases followed by future tax cuts; unilateral and globally coordinated intertemporal redistributions. For (c) I have examined revenue-neutral switches from capital income taxation to consumption taxation; from consumption taxation to capital income taxation; unilateral and globally coordinated tax reforms.

For each experiment I have investigated alternative specifications and assumptions, of which there are several to consider: (i) type of goverment ('exogenous', national planner, global planner); (ii) nature of returns to scale (constant over all inputs, domestic inputs, or private inputs) and associated issue of whether public capital is charged on a user fee basis, rented, or provided for free; (iii) whether an externality is present or absent, and if present, whether the externality is symmetric (countries find own public capital and spillover capital equally productive) or asymmetric (countries find own public capital more productive for own country production); (iv) parameter values and initial conditions - for example, the value of the intertemporal elasticity of substitution ($1/\sigma$), the rates of time preference, the transfer criterion condition, and initial debtor/creditor status (ie. value of initial z).

The experiments and specification tests were all conducted and I shall report on a select few, along with comments as to how the results would be affected, if at all, by changes in specification. Appendix II has the necessary simulation details. The nonlinear model given in Table 1 is linearized around a steady-state consistent with the assumed parameter values and initial conditions. For each policy experiment, the initial and final steady-states will be compared, and the time paths between the two steady-states will be plotted.

The qualitative results do not depend on the initial value of z. The nature of the time paths and the ratios of final to initial steady-state values are similar whether $z = 0$ or nonzero. For this reason I assume initial $z = 0$. I also assume initial $p = 1$. Furthermore, I report the results of only case 1 of the transfer criterion (ie. $(1-\alpha)\theta + \alpha^*\theta^* > r$, where initial $r = 0.05$). The simulation results are consistent with the predictions of whichever configuration of the transfer criterion is assumed. I have also experimented with $\sigma = \sigma^* = 1.5$ and $\sigma = \sigma^* = 0.5$ and have found movements in θ, θ^* to be quite negligible[1]. I therefore assume $\sigma = \sigma^* = 1$ which implies that θ, θ^* are constant (independent of interest rates).

The output elasticities for the production function range as follows: under CRTS across all inputs, $\beta_G = 0.1$, $\beta_{G^*} = 0.1$, $\beta_K = 0.2$, $\beta_L = 0.6$; under CRTS across domestic inputs, $\beta_G = 0.1$, $\beta_{G^*} = 0.1$, $\beta_K = 0.3$, $\beta_L = 0.6$; under CRTS across private inputs, $\beta_G = 0.1$, $\beta_{G^*} = 0.1$, $\beta_K = 0.35$, $\beta_L = 0.65$; and likewise for the foreign production function.

(a) Public Spending Shocks

<u>(i) Permanent, Unanticipated Balanced Budget Increase in i_g.</u>

Consider a 25% increase in domestic long run public investment above its initial steady-state level, financed by higher long run lump sum taxes. Assume there are CRTS across all private inputs and no distorting taxes. In the long run, both home and foreign private capital stocks are higher (up 7.22% from its initial steady-state level), while only the home country's stock of public capital is 25% higher. Long run production is increased by 7.21% in both countries. Long run private consumption is higher in both countries, though relatively higher in the foreign country.

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[1] This may be different in non-linear simulations.

This last outcome has to do with the foreign economy not sharing any burden, in the long run or short run, of financing the additional creation of G. With the lump-sum tax feedback rule in place, home taxes initially rise on impact above their (higher) long run levels. Domestic human wealth falls on impact - hence the short run decrease in home consumption. Gradually both h and c rise as the effects of a higher G affect production and labour income. Foreign wealth rises on impact in response to news of a higher future stream of income.

The long run stock of domestic government debt is lower, since the long run stock of G is higher. There is some movement in the foreign stock of government debt b^* and foreign taxes τ^* along the transition; this has to do with the interest rate being endogenous. As it changes so too do government interest payments. In the long run the domestic terms of trade appreciates, since there is a relatively higher long run demand for domestic output. The reason for this is that while consumption and private investment in both countries are higher, only domestic public investment is higher, so that steady-state domestic absorption ends up being relatively higher than that abroad. In the short run also, domestic absorption is higher because of greater domestic investment at home, causing the home economy to run current account deficits along the transition. In the long run, the foreign economy is the net creditor and has the relatively higher steady-state level of private consumption. The time paths for this experiment are given in Figure 2.8.

I now comment on the results of other specifications. When the returns to scale are changed there are no qualitative changes in the time paths, but there are quantitative changes in the final steady-state values. The general rule has been that the larger the aggregate returns to scale, the larger the final impacts on output. For example when CRTS prevails across domestic inputs, long run foreign and domestic stocks of private capital increase by 6.17% of their initial steady-state levels and long run output in each country rises by 6.54%. These increases are less than the increases under CRTS across private inputs. When CRTS prevails across all inputs, long run private capital stocks rise by 3.67% and long run output in each country rises by 4%. One rather noticeable outcome in this last case is that home consumption actually falls in the long run. That is, with CRTS across all inputs, long run output does not rise sufficiently to raise long run labour income to offset the higher long run financing burden of increased public capital. (The foreign economy does, however, consume more in the long run, and is a free-rider). There must consequently be sufficient scale economies at the aggregate level to make public investment worthwhile for the home economy.

When governments either lend their stock of capital or charge a user fee for capital services, the long run stock of government debt increases in both countries. The reason is that as G increases, the resulting output increases induce fiscal revenues. In the long run higher government revenues are associated with a larger stock of government debt.

Returning to the case where CRTS prevails across private inputs, I illustrate the impacts of the same increase in i_g when no spillovers exist. Figure 2.9 plots the time paths. Essentially only the home country stocks of capital and output increase. In the long run the home economy's terms of trade deteriorates as there is a relatively greater increase in the supply of home output. The home economy's long run consumption is higher and is supported by a higher steady-state stock of external assets. However, along the transition to long run the home economy runs current account deficits as absorption is higher (due to the home investment boom). This and the rising foreign terms of trade enable the foreign country to enjoy higher consumption temporarily. On impact domestic consumption and human wealth do not decline despite the higher short run taxes; home production and labour income increase more when there are no spillovers since global savings do not flow to the foreign country to help finance private capital accumulation there. As a result domestic K and y both increase by 10% of their initial steady-state levels.

A case intermediate to this one of no spillovers and spillovers with symmetric public capital output elasticities is the case of asymmetric output elasticities. Here 'own' country public capital stocks are more productive than the spillover public capital stocks. For example, $\beta_{G*}=0.05$, $\beta_G=0.1$ and $\beta^*_{G*}=0.1$, $\beta^*_G=0.05$. The time paths are qualitatively similar (given the same i_g shock), but home private capital, output, and consumption all rise faster and reach higher long run levels than their foreign counterparts. The long run stock of net external assets z is negative, and the home country's terms of trade deteriorates. The latter occurs because the relative supply of home output is larger in the long run, and $z < 0$ because, despite $c > c^*$, the p increase is relatively larger (ie. $rz = (1-\alpha)c - \alpha^*pc^* < 0$).

<u>(ii) Temporary, Immediately Implemented Balanced Budget Increase in i_g.</u>

Figure 2.10 assumes the same experimental conditions underlying Figure 2.8 (ie. CRTS across all private inputs, and spillovers with 'symmetric' output elasticities of G, G*). Time t_0 is the date of implementation, and t_1 is a future date at which the policy is reversed.

In the long run all variables return to their initial steady-state levels. There are no permanent effects. Capital accumulation is followed by decumulation, current account deficits (in the home country) followed by current account surpluses. The home terms of trade first increases on impact in response to a higher demand for domestic output, then depreciates between t_0, t_1 as foreign absorption is higher along this transition phase (primarily because $c^* > c$ during this period). At t_1 there is a discrete cut in public investment and a decline in demand for domestic output, causing the home terms of trade to decrease on impact at t_1. Thereafter the home terms of trade appreciates as domestic absorption is higher during the rest of the transition. The prices of capital q_p, q_p^* take into account the future policy reversal, jumping only at the date of announcement, and by the time the policy reversal takes place at t_1, the prices of capital overshoot their long run equilibrium levels so as to bring about the expectation that they will be rising towards their long run levels.

(iii) Unanticipated, Permanent Deficit-Financed Increase in i_g

The same increase in long run domestic public investment is financed by bonds. The time paths (see Figure 2.11) are strikingly similar to those in the balanced budget case. The reason is that the lump sum tax feedback rule automatically raises current lump sum taxes on impact, just as if the public investment were financed by taxes. The key difference is that the home country's long run lump sum tax burden is unchanged. As a result there is no gap between foreign and home consumption in the long run.

Moreover the final steady-state magnitudes are larger than they were under balanced-budgets: the private stocks of capital are 8% above their initial steady-state levels and supplies of output 7.5% above their initial steady-state levels. The home economy is also a net creditor in the long run despite $c = c^*$. This is attributable to the long run terms of trade improvement for the home country; given a higher steady-state stock of domestic public capital G, unchanged G^*, and $K = K^*$ and $y = y^*$, domestic absorption is on net higher than foreign absorption in the long run. Consequently there is a relatively higher demand for domestic output in the long run[1].

A higher stock of government capital is associated with a smaller stock of government debt. But note that the two countries have the same long run stock of financial wealth and private capital; this implies that, for the home country, net external debt and government debt have moved in the same direction. In particular, z increased (or - z decreased) and b decreased. Thus a long-run deficit financed increase in public capital (which generates spillovers) works to eliminate the twin "debts".

(iv) A Coordinated Balanced-Budget Increase in i_g, i_g^*

In this experiment long run public investments in both countries are increased by 25% of their initial steady-state levels. The result of this simultaneous expansion is to magnify the effects that were generated by a unilateral expansion and to create offsetting changes in the terms of trade and net external asset positions. The stock of private capital in each country increases by 14.4% and output in each country by 14.38% of their respective initial steady-state levels. Global absorption is higher in the long run, necessitating a decline in long run world interest rates.

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[1] Note that it would be harder to make these inferences if domestic capital accumulation involved foreign country output as well as home output.

(v) Public Consumption Shocks c_g:

Domestic long run public consumption is increased by 25% of its initial steady-state level. The balanced-budget case is shown in Figure 2.12 and deficit-financed case in Figure 2.13. The primary purpose of this set of experiments is to contrast unproductive public spending with productive. The important contrast is that c_g works through "pecuniary" channels of transmission. Under a balanced-budget expansion, interest rates rise, private stocks of capital decumulate, and human wealth and private consumption decline. Foreign consumption however does rise in the long run. This is attributable to the home economy increasing its absorption and running current account deficits along the transition, enabling the foreign country to be a creditor in the long run. This is also mirrored in long run foreign financial wealth being larger than domestic financial wealth. In the long run the home country's terms of trade appreciates as the higher long run c_g makes domestic absorption relatively higher than that abroad. This depends on the home country's public consumption purchases falling on domestic output only.

Under a deficit financed expansion in home public consumption, the lump sum tax feedback rule has the effect of raising taxes in the short run and generating government surpluses along the transition. The long run stock of domestic government debt is lower and thus long run interest rates are lower. This results also in a very modest (1%) increase in long run private capital in each country. Total financial wealth falls for both countries, however. In the home country this arises because government bond holdings are lower; in the foreign country this arises because its external asset holdings are lower. Thus both home and foreign long run private consumption decline.

The reason long run z rises is that, along the transition, the home consumption path lies below the foreign consumption path, since the home country is undergoing a period of higher taxes, and foreigners are benefitting from higher financial wealth (due to higher $q_p{}^*K^*$) and human wealth (due to higher expected labour income and lower interest rates). The home country thus runs current account surpluses. In the long run, since domestic and foreign financial wealth holdings are equal, and since the return of domestic τ to τ_0 induces human wealth stocks to converge, long run private consumption in each country is the same. Since home c_g is higher, home steady-state absorption is higher, and thus the home country's terms of trade is also higher in the long run.

(b) Intertemporal Redistributions of Lump Sum Taxes

In the remaining group of experiments I will focus attention on endogenous government behaviour. Public investments, i_g and $i_g{}^*$, are now endogenous. To alter the stocks of public sector capital, governments will have to turn to other 'exogenous' policy instruments. They are: (1) lump sum taxes; (2) distortionary taxes (consumption taxes or capital income taxes). This section focuses on (1) and the next subsection (c) will deal with (2). Public consumption in each country is held fixed throughout. I also assume CRTS across private inputs and that spillovers are present.

(i) Unanticipated, Permanent Increase in Long Run Taxes τ_0.

Suppose the home country unilaterally increases τ_0 by 25% of its previous steady-state level. In order for long run future taxes to be higher there must be transitorily lower short run taxes and a rise in budgetary deficits which translate into a higher stock of long run government debt, the higher long run interest payments on which need to be financed by higher steady-state taxes. The lump sum taxation feedback rule governs this intertemporal redistributive shift of the burden of taxation to the future.

This experiment was performed on all three types of governments: exogenous (for comparison), national planner, and global planner. The time paths of the variables under each type of government were qualitatively similar, though quantitatively, the final steady-state magnitudes tended to differ, which I will later discuss. A representative example for the case of a national planner is shown in Figure 2.14[1].

In the long run the stock of home country government debt is permanently higher, leading to higher long run world interest rates.

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[1] In this instance there were no qualitative differences in results over how governments provided public capital - ie. freely or on a rental/user fee basis. The long run association between the stock of public capital G and government debt b is the same whether the government charges for the use of public capital or not. This is merely due to the parameter values assumed. Recall from section III that a long run positive association between G and b emerges if $F_G > \delta(1 + \frac{\psi\delta}{2})$ - that is, if marginal government revenues from public capital exceed steady-state expenditures needed to maintain public capital. Under our parameter values $\delta(1 + \frac{\psi\delta}{2}) = 0.11 > F_G$ $(= 0.1 \frac{y}{G})$, implying a long run negative relationship between b and G regardless of whether governments charge $\xi = F_G$ or zero.

Private capital stocks are therefore displaced in the long run, as are public capital stocks (not drawn). Public investments and capital stocks react endogenously and decline in order to balance the marginal productivities between private and public capital. Production in both countries also declines. During the transition to long run, the home country's current account goes into deficit as consumers increase their spending in response to lower short run taxes. Foreign consumption also declines since the value of capital (q_p*K*) falls, causing a drop in foreign financial wealth. The private consumption paths are "twisted" eventually as higher domestic taxes take effect and as foreigners gradually gain back wealth from their accumulation of external assets.

(ii) Unanticipated Coordinated Long Run Tax Cuts in τ_0, τ_0*.

The previous policy led to a long run decline in the stocks of public capital in both countries and to a worsening of the net external asset position of the country which unilaterally pursued the policy. Suppose instead the national planners coordinated to do the reverse policy: namely decrease long run lump sum taxes in each country by 25% of their initial steady-state levels. The time paths are given in Figure 2.15.

To achieve lower long run taxes, both economies must undergo a transitional period of budgetary surpluses. In the long run public debt stocks decline as do world interest rates. Both public capital and private capital increase in the long run. There are no changes in the terms of trade or in current account balances, owing to offsetting pressures. Consumption falls on impact as human wealth is lower (since short run taxes are raised). In the long run taxes decline, human wealth recovers, output increases, and long run consumption increases. If the home country were to implement this long run tax cut policy unilaterally, only its long run private consumption would be higher; long run foreign private consumption would be crowded out as the home country would gain external assets in the long run.

I now discuss the sensitivity of the results to the type of government assumed. There are no qualitative differences but there are quantitative. Typically if a fiscal policy change is expansionary (such as a long run tax cut), the global planner economy produces the greatest increases in long run output and consumption, secondly the national planner economy, and thirdly the exogenous government economy. Conversely if a contractionary fiscal policy is pursued (such as an increase in long run taxes), the global planner economy produces the greatest contraction in long run output and decline in consumption, secondly the national planner economy, and thirdly, the exogenous government economy. Another interpretation of this is that to achieve a given amount or degree of "contraction", the global planner needs only to raise a smaller amount of long run taxes than does a national planner, who in turn needs only to raise a smaller amount of long run taxes than does an exogenously behaving government. To achieve a given amount or degree of expansion, the global planner needs only to reduce a smaller amount of long run taxes than does a national planner, who in turn needs only to reduce a smaller amount of long run taxes than does an exogenously behaving government.

The intuition is that in the exogenous government economy, public investment does not react to changes in the world interest rate or to differences in the marginal productivities of public and private capital. It is the economy that is the least responsive to "fundamentals". The global planner economy is the most responsive; resources are shifted constantly and thoroughly so as to preserve global efficiency conditions at all times.

To provide an idea of how expansionary or contractionary the same policy changes can be under different types of government, I report the relative magnitudes of the final steady-state impacts in Tables 2 and 3. Table 2 refers to the experiment described earlier where the home country unilaterally raises long run taxes. Each entry is the ratio of the final steady-state value of a variable in question to its initial steady-state value. Table 3 refers to the experiment where both countries simultaneously lower long run taxes. In Table 2, the global planner creates the most contraction for the same lump-sum tax increase. Should the global planner want to achieve the same contraction shown in the exogenous government case, it could judiciously chose a lower long run tax increase. In Table 3, the global planner produces the most expansion for a given policy change compared to the other types of government.

(iii) Pursuit of 'Asymmetric' Redistribution of Lump Sum Taxes

In this experiment the home country raises long run taxes while the foreign country reduces long run taxes by the same amount. Figure 2.16 depicts the time paths. This fiscal policy programme neutralizes each country's effect on interest rates. Thus the prices of capital, stocks of capital, and output levels do not move. Movements only occur in consumption related variables. In the short run, domestic human wealth rises (since short run domestic taxes are lower), causing home consumption to increase and home current account deficits to be triggered, while the opposite takes place in the foreign country. In the long run the foreign economy is a net creditor and gets to enjoy greater long run consumption.

(c) Revenue-Neutral Tax Reforms

The experiments here are designed to see how endogenous public capital formation responds to variations in distortionary taxes. In particular I look at revenue neutral switches from capital income taxation to consumption taxation, and vice versa. The policy change is unanticipated and revenue-neutral with reference to the initial steady-state; as time progresses, tax bases will change, and the amount of revenue raised will be endogenously determined[1]. The production functions are CRTS across private inputs.

(i) Unilateral Switch from Capital Income Taxation to Consumption Taxation

The home country is assumed to initiate the reform unilaterally. The consequences are depicted in Figure 2.17. The switch stimulates private investment and public investment. Note that capital accumulation also takes place in the foreign economy. This occurs because of 'spillovers'. A higher stock of domestic public capital increases foreign productivity, as G is an input in the foreign production function. This causes q_p^* to increase and stimulate foreign private and public investment. Foreign capital and output growth are not as significant as the home economy's since foreign capital income taxes are still in place.

[1] An alternative experiment would be to fix government debt for all time and let the distortionary tax instrument that is not reduced or eliminated by tax reform to be an endogenous variable.

During the transition, the home country experiences budgetary surpluses as the long run stock of government debt is lower, an outcome that is consistent with a higher long run stock of public capital. In the foreign country the increase in G^* is modest but the increase in c^* and K^* induces higher foreign government (distortionary tax) revenues so that long run foreign government debt is higher.

During the transition the home economy also runs current account deficits, as domestic absorption is higher (particularly because of the investment boom at home). In the long run $z < 0$, and because there is a relatively greater supply of home goods in the long run, the home country's terms of trade declines in the long run. Had there been no spillovers the home country's terms of trade and external asset position would have deteriorated more. The spillover channel allows foreign absorption to rise somewhat along the transition and cushion the adverse impacts on the home country's terms of trade and balance of payments.

(ii) Unilateral Switch from Consumption Taxation to Capital Income Taxation

Again the home country unilaterally undertakes the reform. Figure 2.18 depicts the time paths. This time public and private capital stocks decumulate. The home country temporarily enjoys a 'consumption boom' because of the consumption tax cut. Again because of the presence of spillovers, the stocks of foreign public and private capital also decline, but not by as much. Thus along the transition, the home country's decline in absorption is greater than that of the foreign country. Consequently the home country runs current account surpluses and becomes a long run creditor; its terms of trade appreciates in the long run because of the relatively shorter supply of domestic output in the long run.

Domestic government debt is higher in the long run since the stock of domestic public capital is lower. The stock of foreign government debt is lower since foreign government revenues (from distortionary taxes) are lower, owing to the decreases in c^* and K^*.

(iii) Globally Coordinated Tax Reforms

Under globally coordinated tax reforms the impacts on net external assets and terms of trade cancel. Figures 2.19, 2.20 show the global planner's coordinated version of the revenue-neutral tax reforms. The time paths of all the variables are identical for each country and the variables mirror the solid curves shown in the 'unilateral' reform cases (ie. Figures 2.17, 2.18). The impact effects, however, are larger under coordination. For example, the decline in world interest rates following a global switch to consumption taxation is greater than the decline following a unilateral switch. Similarly the rise in world interest rates following a global switch to capital income taxation is greater than the rise following a unilateral switch.

VI. Conclusion

This paper has studied public investment and technological externalities arising from public sector capital in the context of an open-economy. Both the small open-economy and two-country models were used for illustration. Government investment decisions were either specified exogenously or derived optimally within a utilitarian social planner framework. A government could either provide the services of public capital for free or, if private factors of production do not fully exhaust national output, charge an efficient user fee or lend its capital to the private sector. A global planner could devise a system of international side payments which internalizes the cross-country spillovers from public capital.

Incorporating public investment has revealed a number of contrasts with 'unproductive' government spending. In a small open-economy a public consumption shock lowers long run private consumption and has an uncertain effect on external assets. A public investment shock, however, increases long run private consumption, although the effect on external assets is still ambiguous. Public investment works to augment supply, incomes, and lifetime personal wealth.

In a two-country model an increase in home country public consumption, however financed, lowers long run home private consumption; if tax-financed, long run foreign private consumption can rise since this policy turns the domestic economy into a long run debtor. On the other hand, an increase in the long run stock of home country public capital raises private consumption and crowds in private capital in both countries (provided spillovers are present). Long run foreign private consumption, however, tends to be relatively higher since foreigners do not bear any of the burden of financing the increased stock of an essentially global public good. When no spillovers exist, the benefits accrue solely to the home country. In fact the gains are much larger to the home country because the supply of global savings do not flow to the foreign country to help finance capital accumulation there as would otherwise occur if spillovers exist to stimulate foreign capital formation. An important qualification to the public capital expansion results is that steady-state depreciation expenses must not be so large as to absorb a large share of steady-state resources. An increase in long run output is insufficient to make an investment in the long run worthwhile if the rise in steady-state expenses necessary to maintain the larger stocks of capital should leave less available for private consumption.

When public investment is endogenous, optimizing governments influence the growth of public capital through changes in tax policy. For example, globally coordinated decreases in long run lump sum taxes stimulate public and private capital accumulation worldwide, and raise long run private consumption in both countries. If only distortionary taxes exist, either a unilaterally implemented or globally coordinated conversion to consumption taxation crowds in public and private capital, and raises long run private consumption in both countries.

In this paper I have tended to focus on private consumption more than on international competitiveness and external balances. While the consequences for the current account and terms of trade are vital, they are somewhat incidental to what happens to consumption in this model. For instance, it is possible for a country to be a long run debtor, experience a long run terms of trade loss, and still have a higher long run steady-state level of consumption than that of another country[1]. This occurs if the country's long run output supply is relatively larger and the country must

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[1] For a steady-state debtor, $rz = (1-\alpha)c - p\alpha^*c^* < 0$ and $z < 0$, or $(1-\alpha)c < p\alpha^*c^*$. If $\alpha^*p > (1-\alpha)$, it easy to find values for c, c* such that $c > c^*$; for example, let $\alpha^*=(1-\alpha)$, $p=1.5$, $c=100$ and $c^*=90$.

undergo current account deficits (increased absorption) along the transition to achieve that state. The point is that while developments pertaining to the current account and terms of trade do often determine who enjoys the higher long run consumption, they do not always. For this reason I have viewed them as "means, not ends".

Several extensions come to mind. First, distortionary taxes have been treated exogenously. The social planner framework could be modified to allow governments to choose tax rates, as well as spending, optimally. A second extension is to consider optimal fiscal policy design under 'uncertainty'. A third is to add more countries. This would allow certain asymmetric features to be modelled, such as a case where some countries generate spillovers but enjoy no spillins, and vice versa. How these several countries might share the burden of financing global public goods should also be raised, and related to international tax harmonization. The local public finance literature on 'tax exporting' and public goods spillovers across communities may provide valuable insights at the international level. A fourth extension is to bring back "money", nominal rigidities, and nominal exchange rates, and reexamine the short run, if not long run, properties of the model. A fifth is to carry out a strategic differential game analysis using the endogenous government framework developed here. Learning behaviour, information sets, and game-equilibrium concepts, remain to be specified. Finally, the paper worked with linearized versions of the non-linear model; a next step would be to try some nonlinear approaches.

Table 1: Summary of Two-Country Model

Home Economy

- Firm Sector -

(i) $Y = F(G^*, G, K, L)$

(ii) $\dot{K} = i_p - \delta K$

(iii) $i_p = \frac{1}{\psi}[q_p - 1]K$

(iv) $\dot{q}_p = (r + \delta)q_p - (1-\tau_K)F_K - \frac{\psi}{2}\left(\frac{i_p}{K}\right)^2$

- Household Sector -

(v) $(1+\tau_c)c = \theta[a + h]$

(vi) $c_H = \alpha c$

(vii) $pc_F = (1-\alpha)c$

(viii) $a = b + z + q_p K$

(ix) $\dot{a} = ra + F_L - \tau - (1 + \tau_c)c$

(x) $\dot{h} = (r+\lambda)h + \tau - F_L$

(xi) $\dot{\theta} = \theta^2 + \theta\, [\frac{1}{\sigma}(\,(1 - \sigma)(r + (\alpha-1)\frac{\dot{p}}{p}) - \rho - \sigma\lambda)\,]$

- Government Sector -

(xii) $\dot{G} = i_g - \delta G$

(xiii) $\dot{a}_g = ra_g + \tau + \tau_c c + \tau_K F_K K - c_g$

where $a_g = q_g G - b$

Table 1 continued

(xiv) (a) i_g exogenous (Exogenous Government)

(b) $i_g = \frac{1}{\psi}[q_g - 1]G$ (Endogenous Government)

where

$$\dot{q}_g = (r + \delta)q_g - F_G - \frac{\psi}{2}\left(\frac{i_g}{G}\right)^2 \quad \text{if National Planner}$$

$$\dot{q}_g = (r + \delta)q_g - F_G - pF^*_G - \frac{\psi}{2}\left(\frac{i_g}{G}\right)^2 \quad \text{if Global Planner}$$

(xv) $\tau = \tau_0 + \eta\dot{b}, \quad \eta < -1$

- External Balance -

(xvi) $\dot{z} = rz + c_H^* - pc_F$

(xvii) $y = c_H + c_H^* + c_g + J_p + J_g$

where $J_p = i_p + \frac{\psi}{2}\left(\frac{i_p^2}{K}\right), \quad J_g = i_g + \frac{\psi}{2}\left(\frac{i_g^2}{G}\right)$

Foreign Economy

- Firm Sector -

(i)' $Y^* = F^*(G, G^*, K^*, L^*)$

(ii)' $\dot{K}^* = i_p^* - \delta^* K^*$

(iii)' $i_p^* = \frac{1}{\psi^*}[q_p^* - 1]K^*$

Table 1 continued

(iv)' $\dot{q}_p^* = (r^* + \delta^*)q_p^* - (1-\tau_K^*)F^*_{K^*} - \frac{\psi^*}{2}\left(\frac{i_p^*}{K^*}\right)^2$

- Household Sector -

(v)' $(1+\tau_c^*)c^* = \theta^*[a^* + h^*]$

(vi)' $c_H^* = \alpha^* p c^*$

(vii)' $c_F^* = (1-\alpha^*)c^*$

(viii)' $a^* = b^* + z^* + q_p^* K^*$

(ix)' $\dot{a}^* = r^* a^* + F^*_{L^*} - \tau^* - (1 + \tau_c^*)c^*$

(x)' $\dot{h}^* = (r^*+\lambda^*)h^* + \tau^* - F^*_{L^*}$

(xi)' $\dot{\theta}^* = \theta^{*2} + \theta^* [\frac{1}{\sigma^*} ((1 - \sigma^*)(r^* + \alpha^*\frac{\dot{p}}{p}) - \rho^* - \sigma^*\lambda^*)]$

- Government Sector -

(xii)' $\dot{G}^* = i_g^* - \delta^* G^*$

(xiii)' $\dot{a}_g^* = r^* a_g^* + \tau^* + \tau_c^* c^* + \tau_K^* F^*_{K^*} K^* - c_g^*$

where $a_g^* = q_g^* G^* - b^*$

(xiv)' (a) i_g^* exogenous (Exogenous Government)

(b) $i_g^* = \frac{1}{\psi^*}[q_g^* - 1]G^*$ (Endogenous Government)

Table 1 continued

where

$$\dot{q}_g{}^* = (r^* + \delta^*)q_g{}^* - F^*{}_{G^*} - \frac{\psi^*}{2}\left(\frac{i_g{}^*}{G^*}\right)^2 \qquad \text{if National Planner}$$

$$\dot{q}_g{}^* = (r^* + \delta^*)q_g{}^* - F_{G^*} - pF^*{}_{G^*} - \frac{\psi^*}{2}\left(\frac{i_g{}^*}{G^*}\right)^2 \qquad \text{if Global Planner}$$

(xv)' $\tau^* = \tau_0{}^* + \eta^*\dot{b}^*, \quad \eta^* < -1$

- External Balance -

(xvi)' $\dot{z}^* = r^*z^* + c_F - \dfrac{c_H{}^*}{p}$

(xvii)' $y^* = c_F + c_F{}^* + c_g{}^* + J_p{}^* + J_g{}^*$

$$\text{where } J_p{}^* = i_p{}^* + \frac{\psi^*}{2}\left(\frac{i_g^{*\,2}}{K}\right), \quad J_g{}^* = i_g{}^* + \frac{\psi^*}{2}\left(\frac{i_g^{*\,2}}{G}\right)$$

<u>Global Linkages</u>

(xviii) $r = r^* + \dfrac{\dot{p}}{p}$

(xix) $z + pz^* = 0$

Table 2. Long Run Tax Increase in Home Country
(ie. τ_0 raised by 25% of initial steady-state level)

Ratio of Final Steady-State Values to Initial

	Exogenous Government	National Planner	Global Planner
K	0.994	0.9896	0.9789
K*	0.994	0.9896	0.9789
y	0.998	0.9943	0.9884
y*	0.998	0.9943	0.9884
c	0.98	0.9759	0.99
c*	1.018	1.0138	0.99

Table 3. Long Run Tax Decrease in Both Countries
(ie. τ_0, τ_0* cut by 25% of initial steady-state level)

Ratio of Final Steady-State Values to Initial

	Exogenous Government	National Planner	Global Planner
K	1.013	1.02	1.04
K*	1.013	1.02	1.04
y	1.005	1.011	1.02
y*	1.005	1.011	1.02
c	1.0024	1.008	1.012
c*	1.0024	1.008	1.012

Appendix I: Aggregation of Individual Welfare Functions

Each country's government is assumed to be a utilitarian planner. The objective functionals for the domestic and foreign national planners are, respectively:

$$\text{(A1)}\quad SW_t = \int_t^{\infty} U(c(s))\, e^{-\gamma(s-t)} ds, \qquad \text{(A1)'}\quad SW^*_t = \int_t^{\infty} U^*(c^*(s))\, e^{-\gamma^*(s-t)} ds$$

where γ, γ^* are the national planners' time preference rates, c(s) and c*(s) aggregate consumption, and U(.) and U*(.) aggregate (instantaneous) welfare[1].

The global planner maximizes:

$$\text{(A2)}\quad GW_t = \omega\, SW_t + (1-\omega)\, SW^*_t$$

where ω, $(1-\omega)$ are the weights the planner attaches to the two economies. In the text I assume that the global planner weights the two countries equally ($\omega = 0.5$) and discounts the welfare of future generations in the two countries at the same rate ($\gamma' = \gamma = \gamma^*$).

This appendix shows how the welfare of agents alive and to-be-born within an economy can be aggregated and expressed by functions such as U(.) and U*(.) above. The derivation works through a sequence of indirect utility functions:

Home Economy

$$\text{(A3)}\quad SW_t = \int_t^{\infty} \{ \int_v^{\infty} u(c(s,v))\, e^{-(\rho + \lambda)(s-v)} ds \}\, e^{-\gamma(v-t)}\, dv$$
$$+ \int_{-\infty}^{t} \{ \int_t^{\infty} u(c(s,v))\, e^{-(\rho + \lambda)(s-v)} ds \}\, e^{-\gamma(v-t)}\, dv$$

---

[1] Aggregate and per capita aggregate magnitudes coincide here since aggregate population has been normalized to unity.

Aggregate social welfare consists of two parts: the first part represents the discounted welfare of agents to be born and the second part represents the discounted welfare of agents already alive. In each part, the inner integral sums welfare over time for each agent of vintage v and the outer integral sums across vintages. Note that welfare for each agent is discounted back to the date of birth and not to calendar time. This is necessary to preserve time-consistency. Otherwise, as time progresses, those previously newly born will be amongst those already alive and will be treated differently from those who become currently newly born[1]. The private agent's time preference rate ρ need not be the same as the planner's γ.

Change the order of integration:

$$\text{(A3)}' \quad SW_t = \int_t^{\infty} \{ \int_{-\infty}^{s} u(c(s,v))\, e^{-(\rho + \lambda)(s-v)} dv \}\, e^{-\gamma(s-t)}\, ds$$

and define:

$$\text{(A4)} \quad U(c(s)) = \max \int_{-\infty}^{s} u(c(s,v))\, e^{-(\rho + \lambda)(s-v)}\, e^{-\gamma(s-v)}\, dv$$

$$\text{s.t.} \quad \text{(A4a)} \quad c(s) = \int_{-\infty}^{s} c(s,v)\, \lambda e^{-\lambda(s-v)} dv$$

$$\text{where} \quad \text{(A4b)} \quad c(s,v) = c_H(s,v) + p(s)\, c_F(s,v)$$

To illustrate, I assume a constant relative risk aversion (CRRA) utility function as in the main text:

$$\text{(A5)} \quad u(c(s,v)) = \max u[c_H(s,v), c_F(s,v)] = \frac{[c_H(s,v)^{\alpha}\, c_F(s,v)^{1-\alpha}]^{1-\sigma}}{1-\sigma}$$

subject to (A4b)

---

1 see also Buiter-Kletzer (1990) and Calvo-Obstfeld (1988).

for each individual v. Solving problem (A5) yields:

$$u(c(s,v)) = \Gamma\ p(s)^{(1-\sigma)(\alpha-1)}\frac{c(s,v)^{1-\sigma}}{1-\sigma} \quad \text{where } \Gamma = [\alpha^{\alpha}1-\alpha^{1-\alpha}]^{1-\sigma}$$

Substituing this into (A4) gives:

$$\text{(A4)'}\quad U(c(s)) = \max \int_{-\infty}^{s} \Gamma\ p(s)^{(1-\sigma)(\alpha-1)}\frac{c(s,v)^{1-\sigma}}{1-\sigma} e^{-(\rho+\lambda)(s-v)} e^{-\gamma(s-v)} dv$$

s.t. (A4a)

The solution to problem (A4)' is:

$$\text{(A6)}\quad \frac{\frac{\partial c(s,v)}{\partial v}}{c(s,v)} = \frac{\rho-\gamma}{\sigma} \quad \text{for } v \in [\,s,\infty)$$

or

$$\text{(A6)'}\quad c(s,v) = c(s,s)e^{(\frac{(\rho-\gamma)}{\sigma})(v-s)}$$

Substituting (A6)' into (A4a) gives:

$$c(s,s) = (\frac{\sigma\lambda+\rho-\gamma}{\sigma\lambda})\ c(s)$$

and substituting this back into (A6)' gives:

$$\text{(A7)}\quad c(s,v) = (\frac{\sigma\lambda+\rho-\gamma}{\sigma\lambda})\ e^{(\frac{(\rho-\gamma)}{\sigma})(v-s)}\ c(s)$$

for which it is required that $\sigma\lambda > \gamma - \rho$, ie. that the discrepancy between the planner's time preference rate and the individual's rate not be "too large".

Substituting (A7) into (A4)' gives:

$$\text{(A8)} \quad U(c(s)) = \Phi \frac{c(s)^{1-\sigma}}{1-\sigma} p(s)^{(1-\sigma)(\alpha-1)}$$

$$\text{where} \quad \Phi = \Gamma \left(\frac{\sigma}{\lambda\sigma + \rho - \gamma}\right)^{\sigma} \lambda^{\sigma-1} \text{ and } \Gamma = [\alpha^{\alpha} 1-\alpha^{1-\alpha}]^{1-\sigma}$$

Finally substituting (A8) into (A1) gives us the home country's national welfare functional, as used in the main text. The condition that ($\sigma\lambda > \gamma - \rho$) ensures that welfare is positive and bounded. Φ will be referred to as the "taste" shifter.

Foreign Economy

The derivations for aggregating foreign individual utilities are similar. I report the final outcome:

$$\text{(A9)} \quad U^*(c^*(s)) = \Phi^* \frac{c^*(s)^{1-\sigma^*}}{1-\sigma^*} p(s)^{(1-\sigma)\alpha^*}$$

$$\text{where} \quad \Phi^* = \Gamma^* \left(\frac{\sigma^*}{\lambda^*\sigma^* + \rho^* - \gamma^*}\right)^{\sigma^*} \lambda^{*\,\sigma^*-1} \text{ and } \Gamma^* = [\alpha^{*\,\alpha^*} 1-\alpha^{*\,1-\alpha^*}]^{1-\sigma^*}$$

Substituting (A9) into (A1)' gives us the foreign country's national welfare functional.

Appendix II: Simulation Model

Let x^p be the predetermined state variables
x^n the non-predetermined state variables
v the output variables
u the exogenous variables.

The state-space form of the model is:

$$\dot{x} = Ax + Bu$$
$$v = Cx + Du$$

where A, B, C, D are matrices of coefficients and

$$x = \begin{bmatrix} x^p \\ x^n \end{bmatrix}$$

Benchmark Model ($\sigma = \sigma^* = 1$; Exogenous Government)

Predetermined state variables, $\mathbf{x^p}$: K, K*, G, G*, z, b, b*
Non-predetermined state variables, $\mathbf{x^n}$: q_p, q_p*, h, h*

Output Variables, **v**: y, y*, c, c*, a, a*, τ, τ*, $\dot{p}$, p, r, r*
Exogenous Variables, **u**: τ_0, τ_0*, i_g, i_g*, c_g, c_g*, τ_k, τ_k*, τ_c, τ_c*.

Notation: (* denotes foreign country variables)

K, K*	private capital stocks	G, G*	public capital stocks
h, h*	human wealth	a, a*	financial wealth
y, y*	output	c, c*	private consumption
b, b*	government debt stocks	τ, τ*	endogenous lump-sum taxes
r, r*	interest rates	τ_0, τ_0*	exogenous lump-sum taxes
i_g, i_g*	public investment	c_g, c_g*	public consumption
τ_k, τ_k*	capital income tax rates	τ_c, τ_c*	consumption tax rates

z net stock of foreign assets (denominated in home output)

q_p, q_p^* ratio of market value of equity capital to replacement cost
p relative price of foreign output in terms of home output
(or reciprocal of the home country's terms of trade)

State Equations:

$$\dot{K} = \frac{1}{\psi}(q_p - 1)K - \delta K$$

$$\dot{K}^* = \frac{1}{\psi^*}(q_p^* - 1)K^* - \delta^* K^*$$

$$\dot{G} = i_g - \delta G$$

$$\dot{G}^* = i_g^* - \delta^* G^*$$

$$\dot{z} = rz + \alpha^* p c^* - (1-\alpha)c$$

$$\dot{b} = rb + i_g + \frac{1}{\psi}\left(\frac{i_g^2}{G}\right) + c_g - \tau - \tau_c c - \tau_k \beta_K y - \xi G$$

$$\dot{b}^* = r^* b^* + i_g^* + \frac{1}{\psi^*}\left(\frac{i_g^{*2}}{G^*}\right) + c_g^* - \tau^* - \tau_c^* c^* - \tau_k^* \beta^*_K y^* - \xi^* G^*$$

$$\dot{q}_p = (r+\delta)q_p - \left[(1-\tau_K)\frac{\beta_K y}{K} + \frac{\psi}{2}\left(\frac{\dot{K}+\delta K}{K}\right)^2\right]$$

$$\dot{q}_p^* = (r^*+\delta^*)q_p^* - \left[(1-\tau_K^*)\frac{\beta_K^* y^*}{K^*} + \frac{\psi^*}{2}\left(\frac{\dot{K}^*+\delta^* K^*}{K^*}\right)^2\right]$$

$$\dot{h} = (r+\lambda)h + \tau - \beta_L y$$

$$\dot{h}^* = (r^*+\lambda^*)h^* + \tau^* - \beta_L^* y^*$$

Output Equations:

$$y = \pi\, G^{*\beta_{G^*}} G^{\beta_G} K^{\beta_K} L^{\beta_l}$$

$$y^* = \pi^*\, G^{\beta^*_G} G^{*\beta^*_{G^*}} K^{*\beta^*_{K^*}} L^{*\beta^*_{L^*}}$$

$$c = \frac{1}{(1+\tau_c)}\theta(a+h)$$

$$c^* = \frac{1}{(1+\tau_c^*)}\theta^*(a^*+h^*)$$

$$a = b + z + q_p K$$

$$a^* = b^* - \frac{z}{p} + q_p^* K^*$$

$$\tau = \tau_0 + \eta\dot{b}$$

$$\tau^* = \tau_0^* + \eta^*\dot{b}^*$$

$$\frac{\dot{p}}{p} = r - r^*$$

$$y = \alpha^* p c^* + \alpha c + c_g + i_g + \frac{\psi i_g^2}{2G} + (\dot{K}+\delta K) + \frac{\psi(\dot{K}+\delta K)^2}{2 \quad K}$$

$$\dot{y} = \frac{d}{dt}\left[\alpha^* p c^* + \alpha c + c_g + i_g + \frac{\psi i_g^2}{2G} + (\dot{K}+\delta K) + \frac{\psi(\dot{K}+\delta K)^2}{2 \quad K}\right]$$

$$\dot{y}^* = \frac{d}{dt}\left[(1-\alpha^*)c^* + \frac{(1-\alpha)}{p}c + c_g^* + i_g^* + \frac{\psi^* i_g^{*2}}{2\ G^*} + (\dot{K}^*+\delta^* K^*) + \frac{\psi^*(\dot{K}^*+\delta^* K^*)^2}{2 \quad K^*}\right]$$

The last two equations are derived by time differentiating the two goods market clearing equilibrium conditions in order to obtain additional equations containing the domestic and foreign interest rates. The interest rates will appear after the appropriate state equations (such as the $\dot{c}$, $\dot{c}^*$ equations) are substituted into those equations.

To simulate this model I linearize the model around an initial stationary-state:

An example:

Parameters: $\beta_G=\beta_{G*}=\beta^*_G=\beta^*_{G*}=0.1$; $\beta_K = \beta^*_K = 0.35$, $\beta_L = \beta^*_L = 0.65$, $\alpha^* = 1 - \alpha = 0.4$, $\alpha = 0.6 = 1 - \alpha^*$, $\psi = \psi^* = 2$, $\delta = \delta^* = 0.1$, $\eta = \eta^* = -2$, $\rho=\rho^*=0.043$, $\lambda = \lambda^* = 0.03$, $\xi = \xi^* = 0$, $\pi=\pi^*=8.8$

Exogenous Values: $\tau_0 = \tau_0^*= 3.114$, $\tau_k = \tau_k^* = \tau_c = \tau_c^*= 0.2$, $i_g = i_g^* = 8.57$, $c_g = c_g^*= 19.88$

These generate:

Steady-State Values: $r=r^*= 0.05$, $p=1$, $z=0$, $q_p=q_p^*=1.2$, $K=K^*=240$, $G=G^*=85.7$, $y=y^*=145.7$, $c=c^*=90$, $\theta=\theta^*=0.073$, $a=a^*=328$, $h=h^*=1145$, $b=b^*=40$, $\tau = \tau^*= 3.114$

Eigenvalues: -0.1045, -0.101, -0.1, -0.1, -0.055, -0.042, -0.025, 0.061, 0.071, 0.18, 0.185.

Associated Settling Times (to reach within 1% of steady-state), respectively: 44.05, 45.64, 46.05, 46.05, 83.5, 110.04, 183.9, -75.4, -64.7, -25.5, -24.8[1].

[1] The eigenvalue with the smallest modulus is associated with net external assets, z. As a result, the current account dynamics in the system are most sluggish. It takes z 183.9 periods to settle down within 1% of steady-state.

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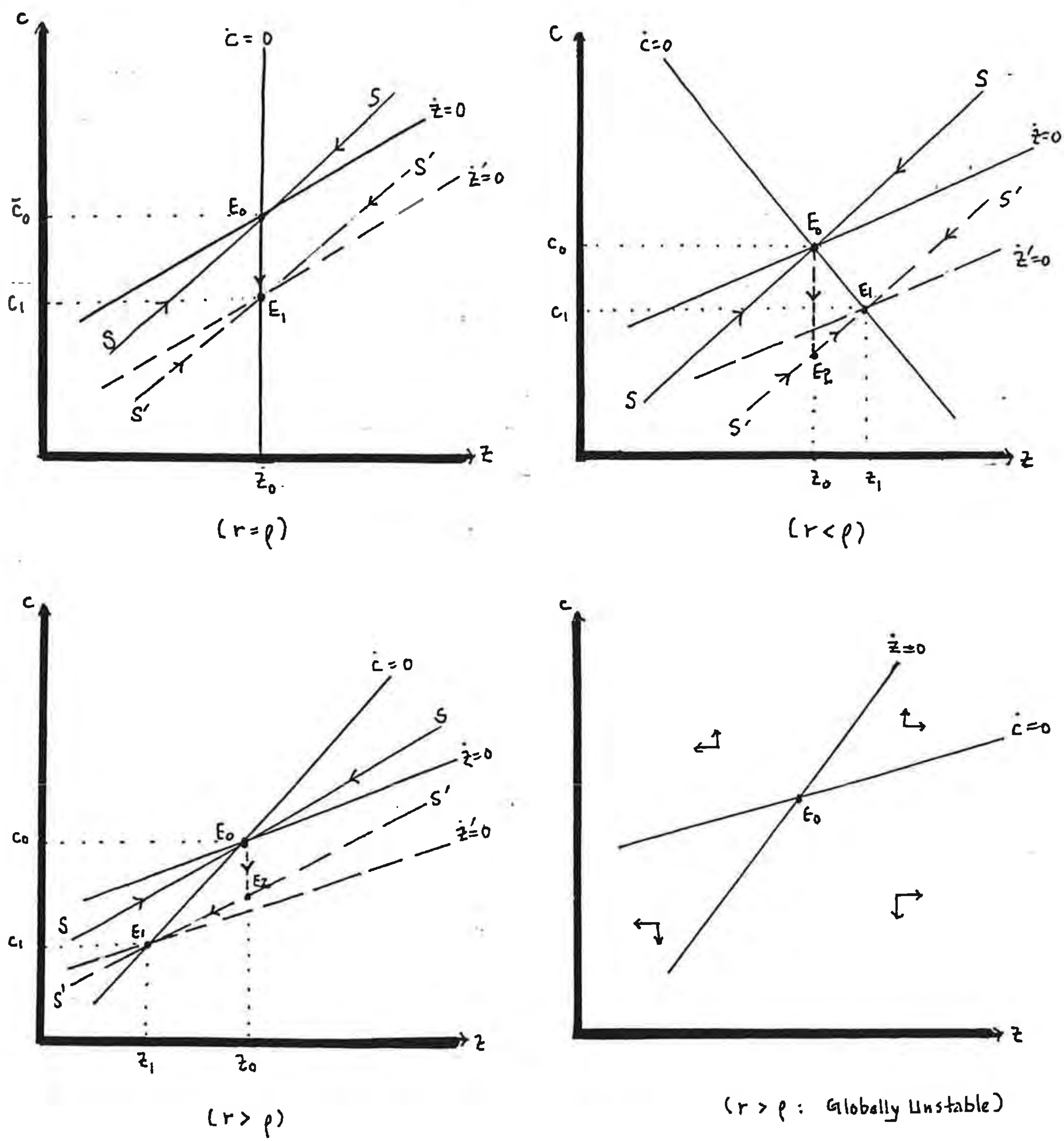


Figure 1: Small Open-Economy Public Consumption Shock

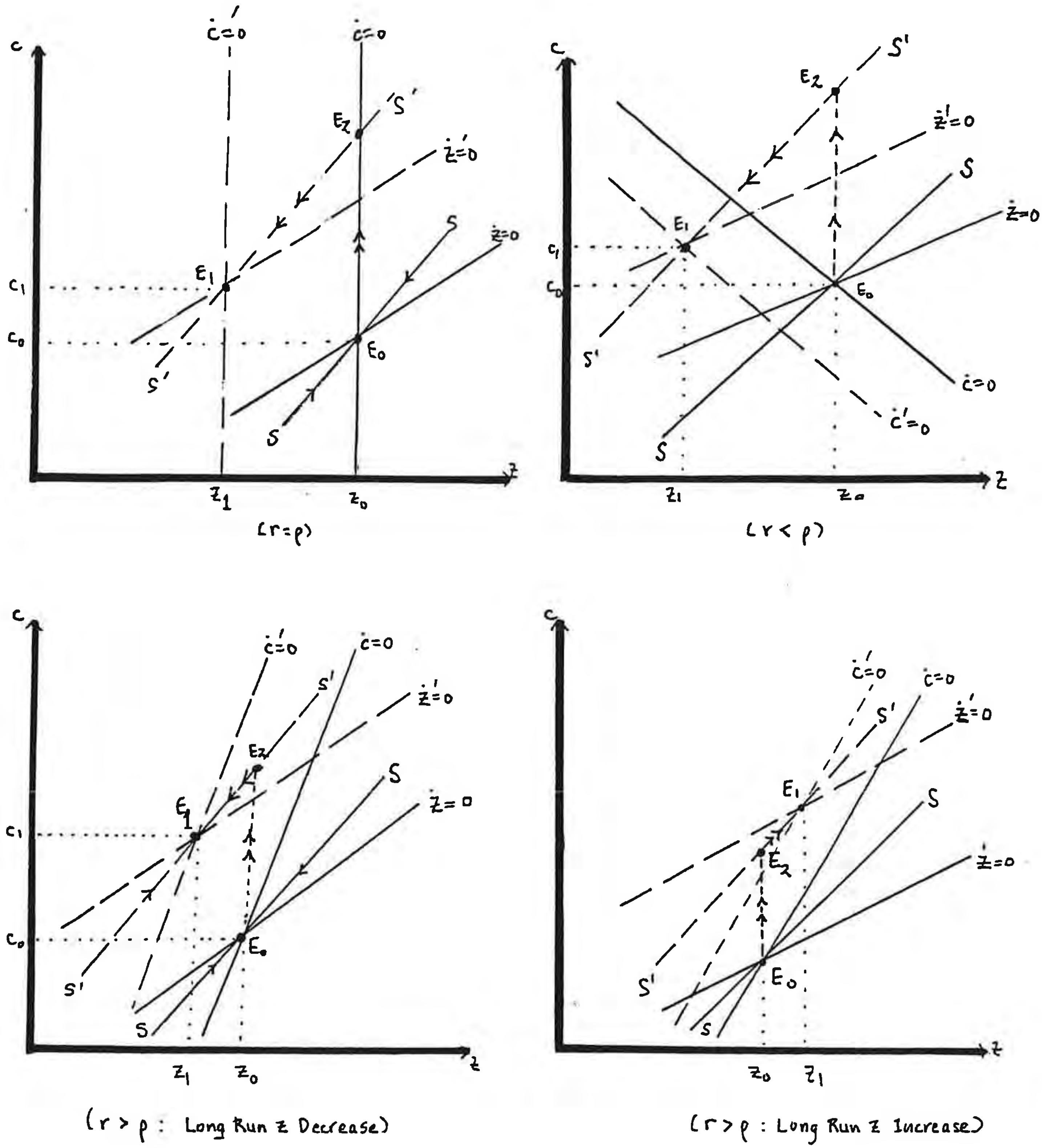


Figure 2 : Small Open-Economy Public Investment Shock

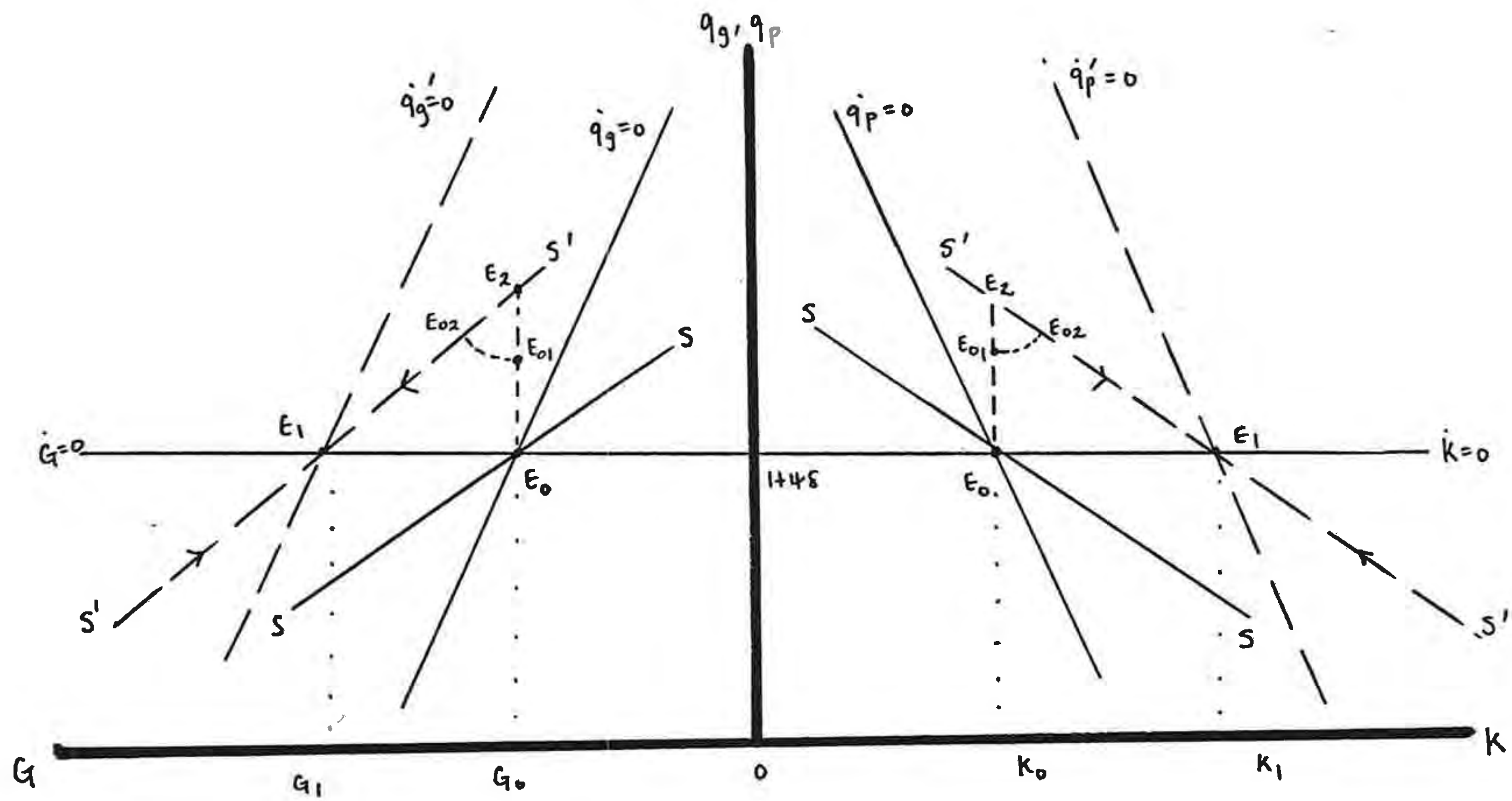
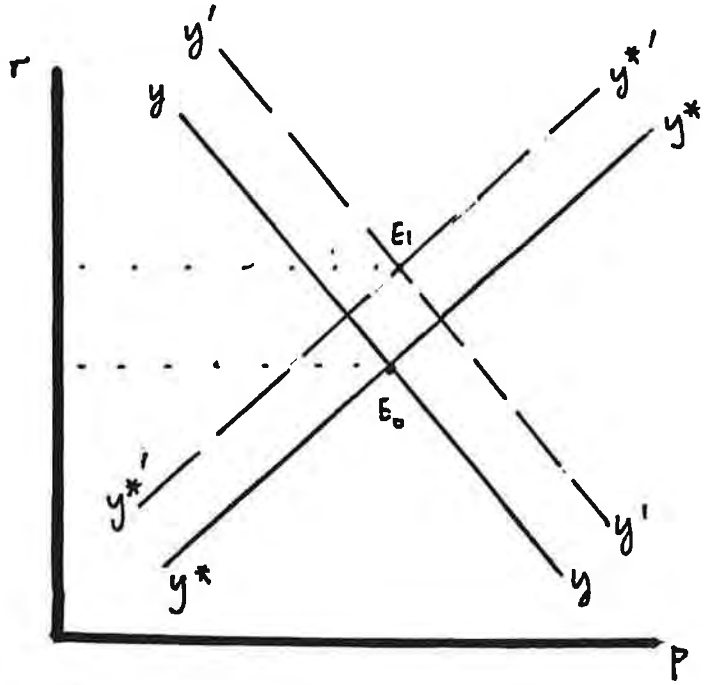
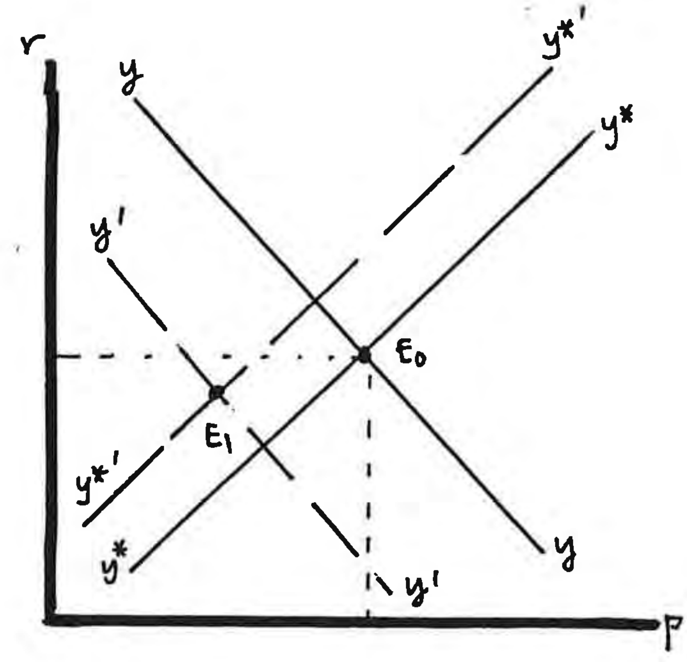


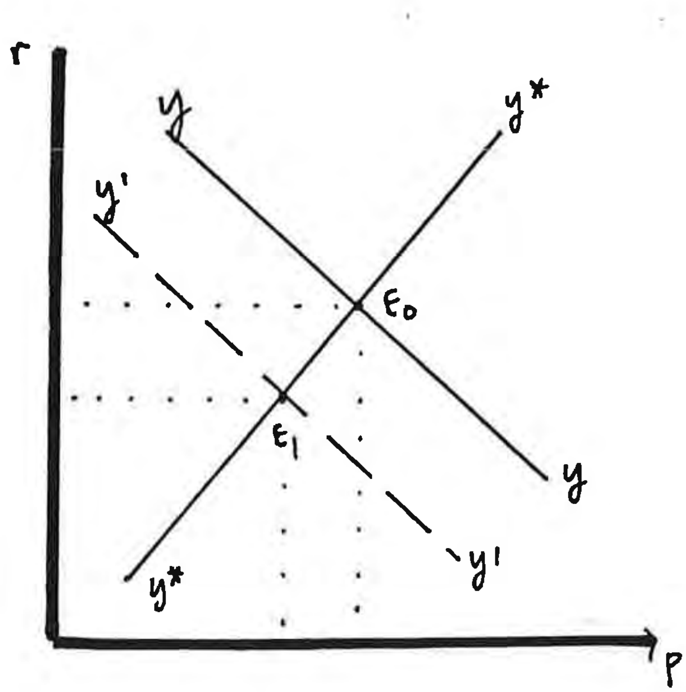
Figure 3': Permanent Increase In Foreign Public Investment



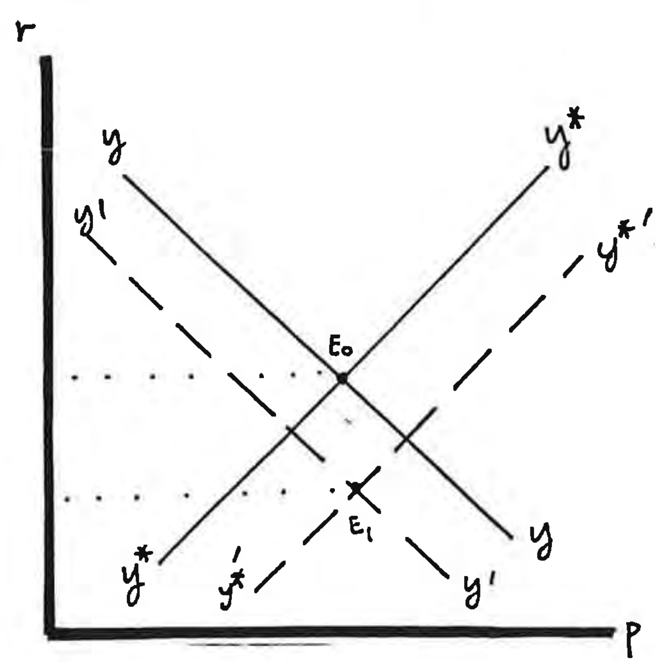
(a) Increase in τ_0



(b) Increase in τ_K ($x_{17} << 0$)



(c) Increase in c_g



(d) Increase in G ($x_{11} < 0, x_{21} < 0$)

Figure 4: Steady-State Impacts on International Prices

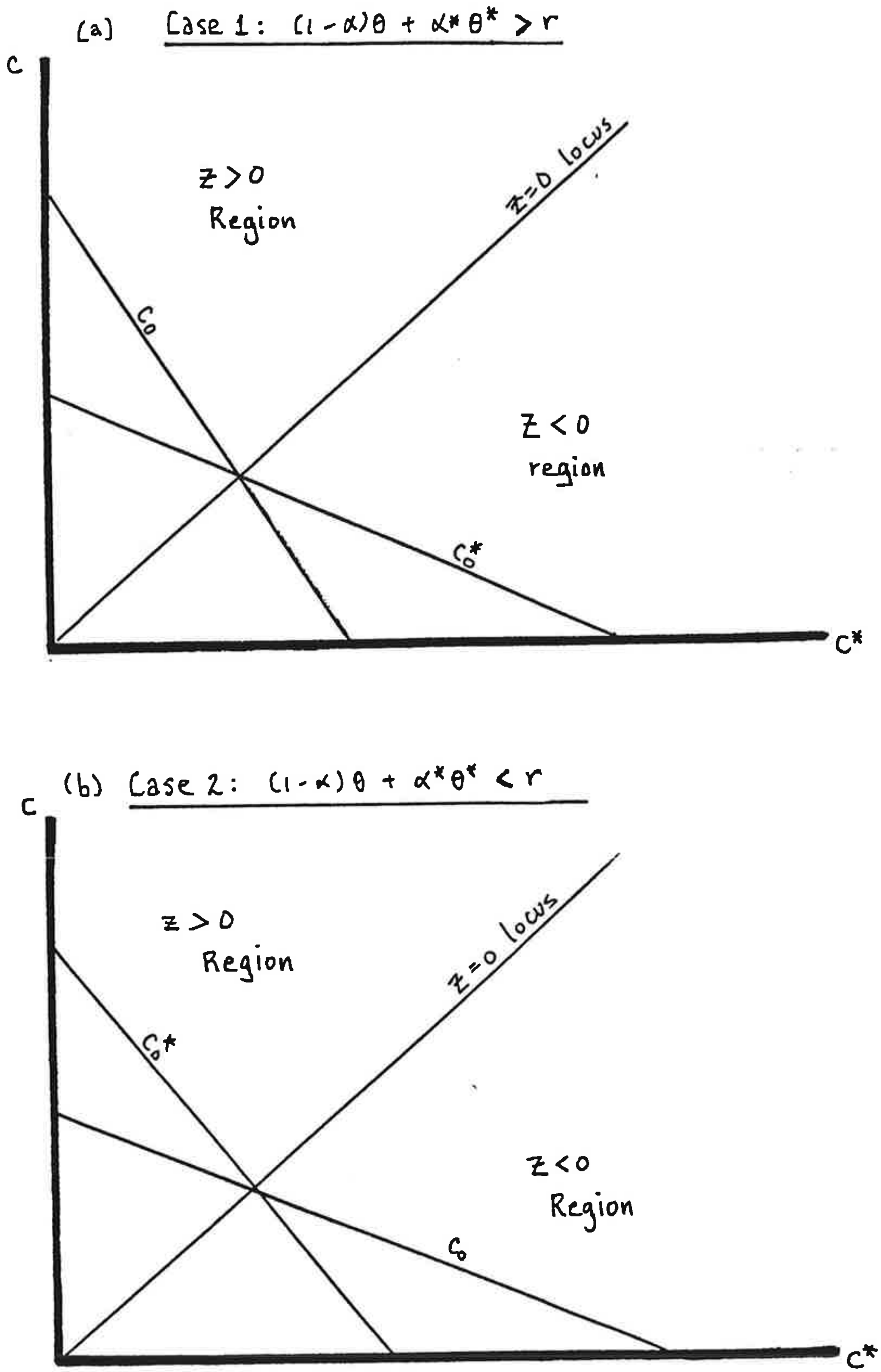


Figure 5: Steady-State Consumption Equilibria Space

Long Run Increase In Domestic Public Consumption c_g

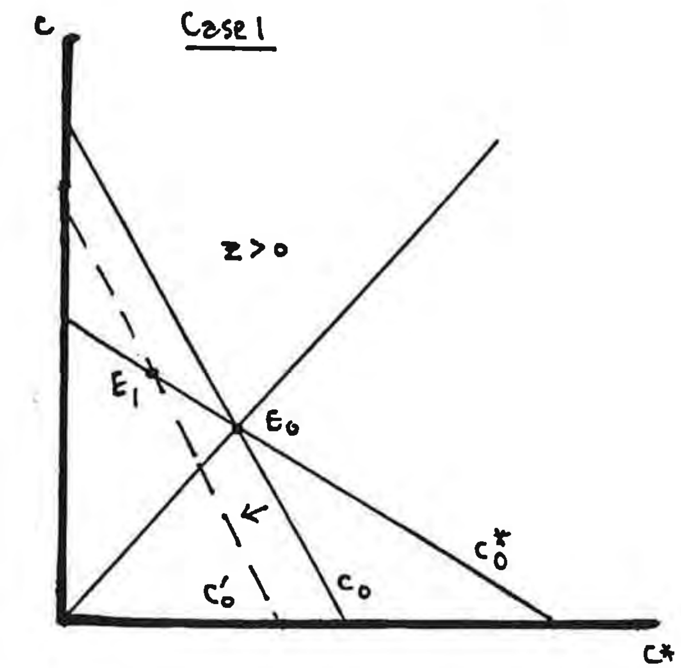


Figure 6a

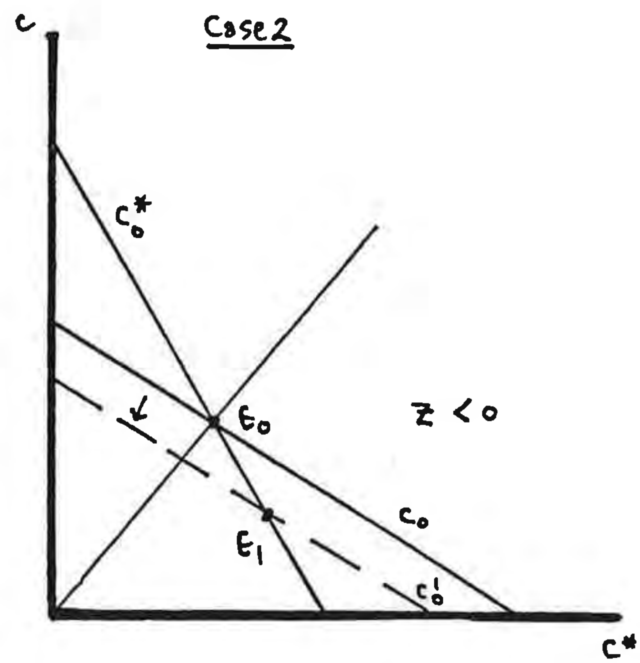


Figure 6b

Long Run Increase In Domestic Public Capital G

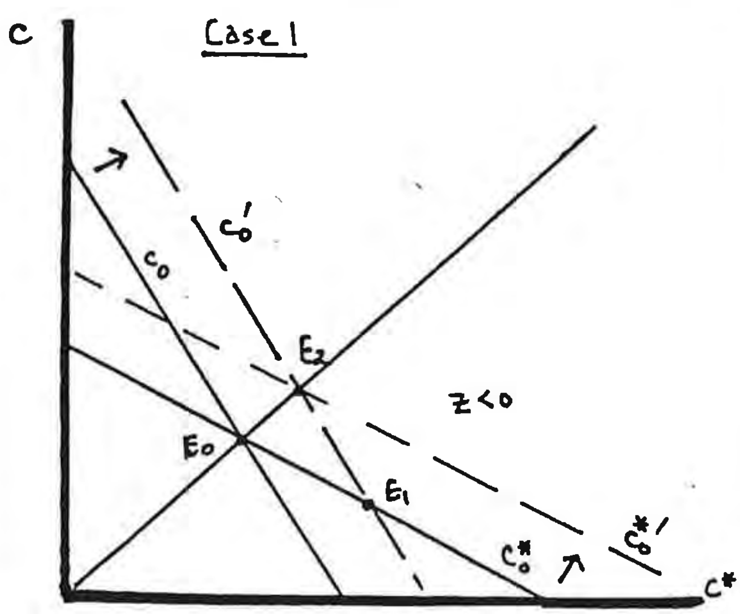


Figure 6c

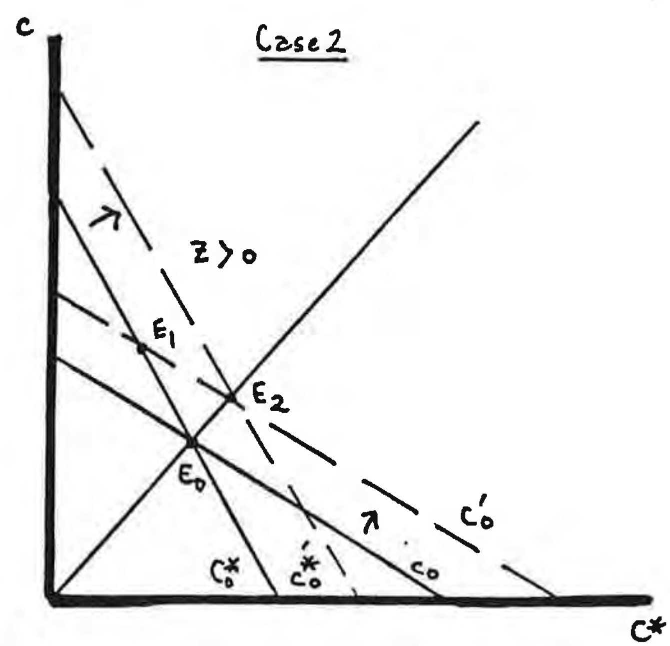


Figure 6d

Long Run Increase in World Interest Rate r

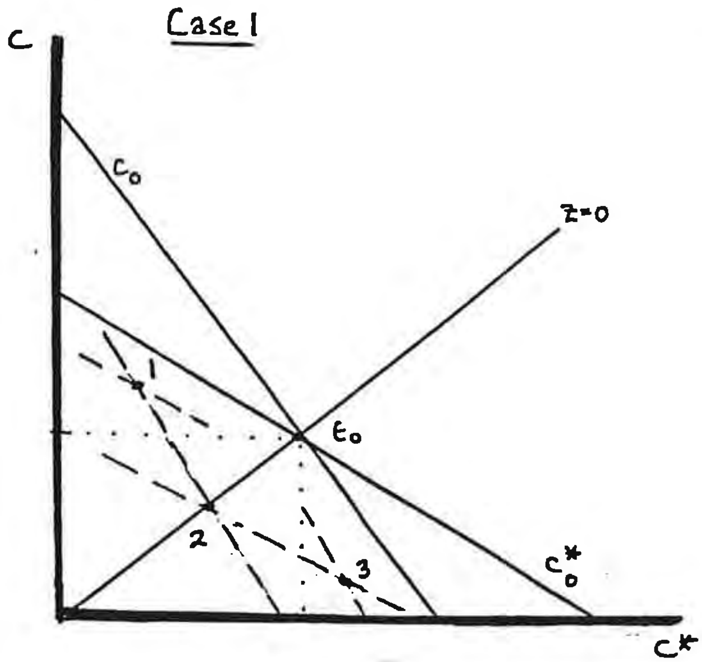


Figure 7a

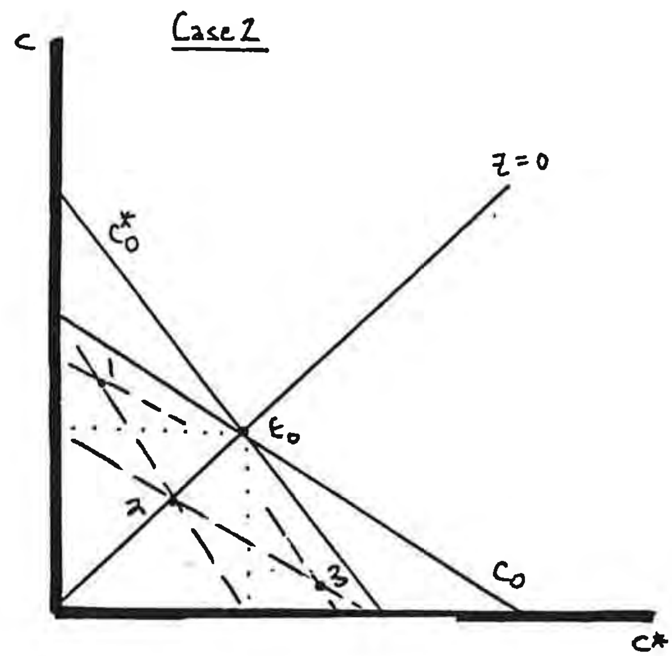


Figure 7b

Long Run Increase in Foreign Terms of Trade p

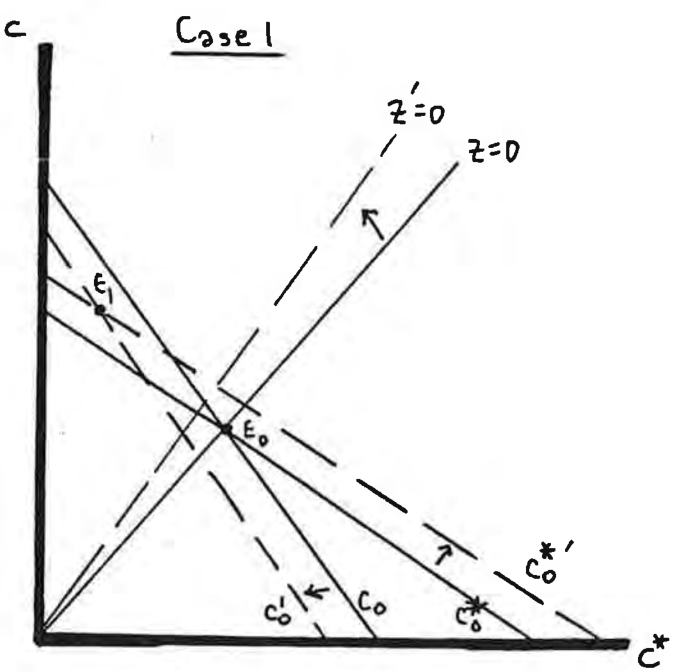


Figure 7c

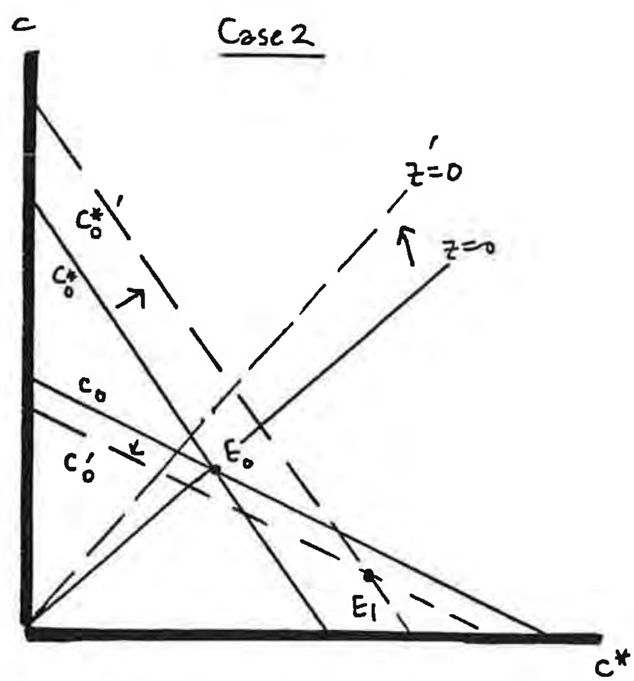


Figure 7d

Figure 8: Unanticipated Permanent Balanced Budget Increase in i_g

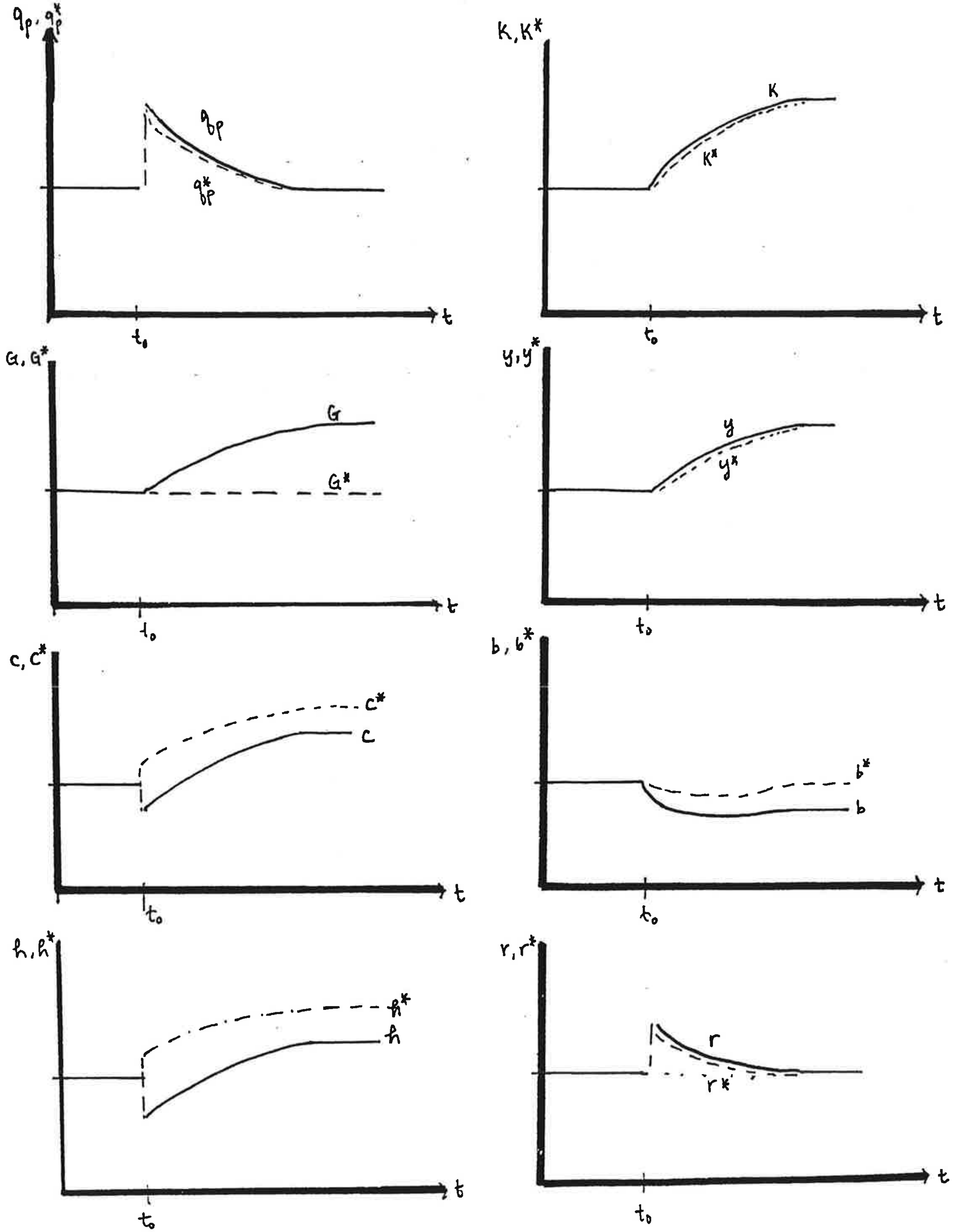


Figure 8 continued

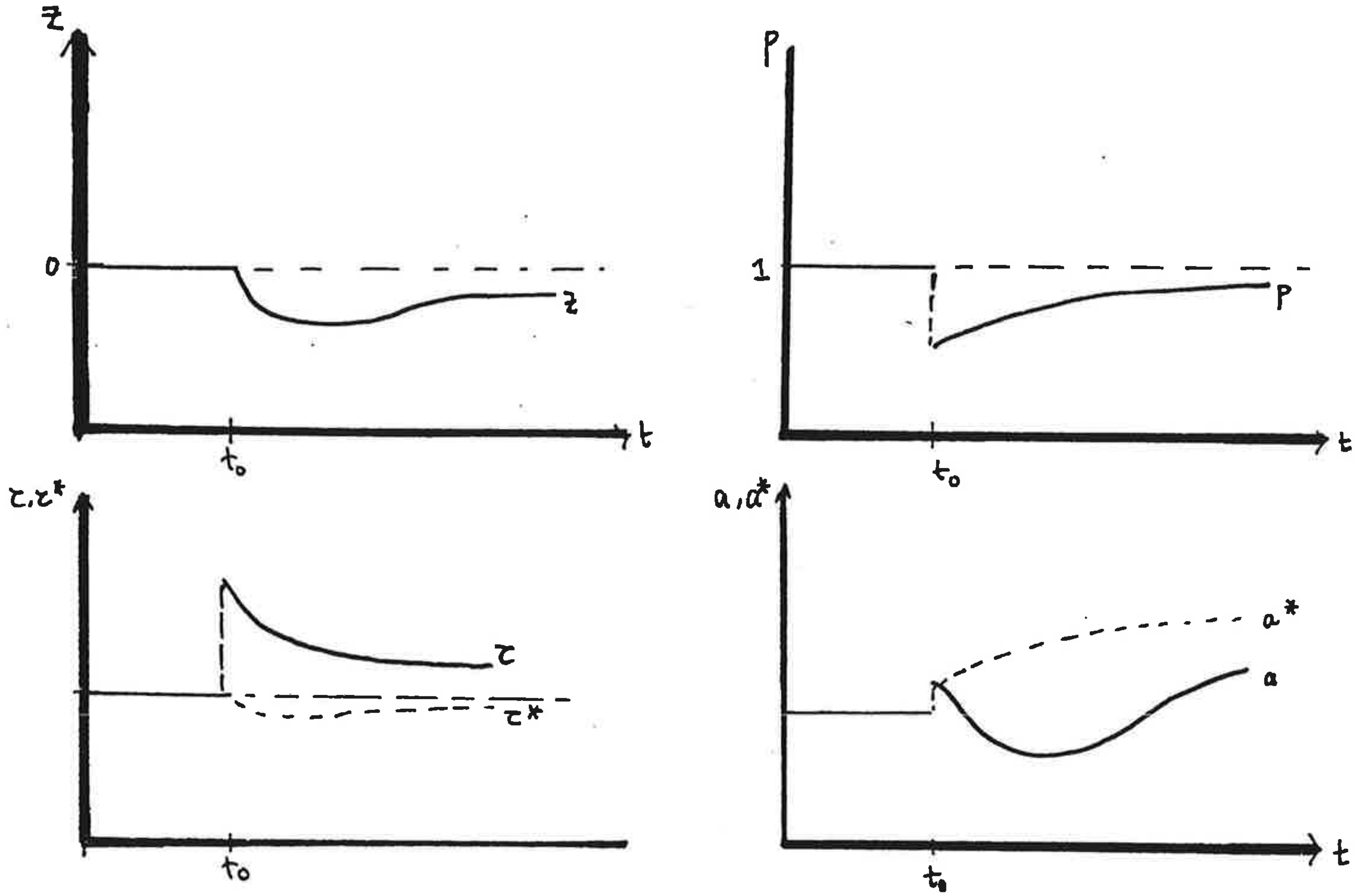


Figure 9; Unanticipated Permanent Balanced Budget Increase in i_g (No Spillovers)

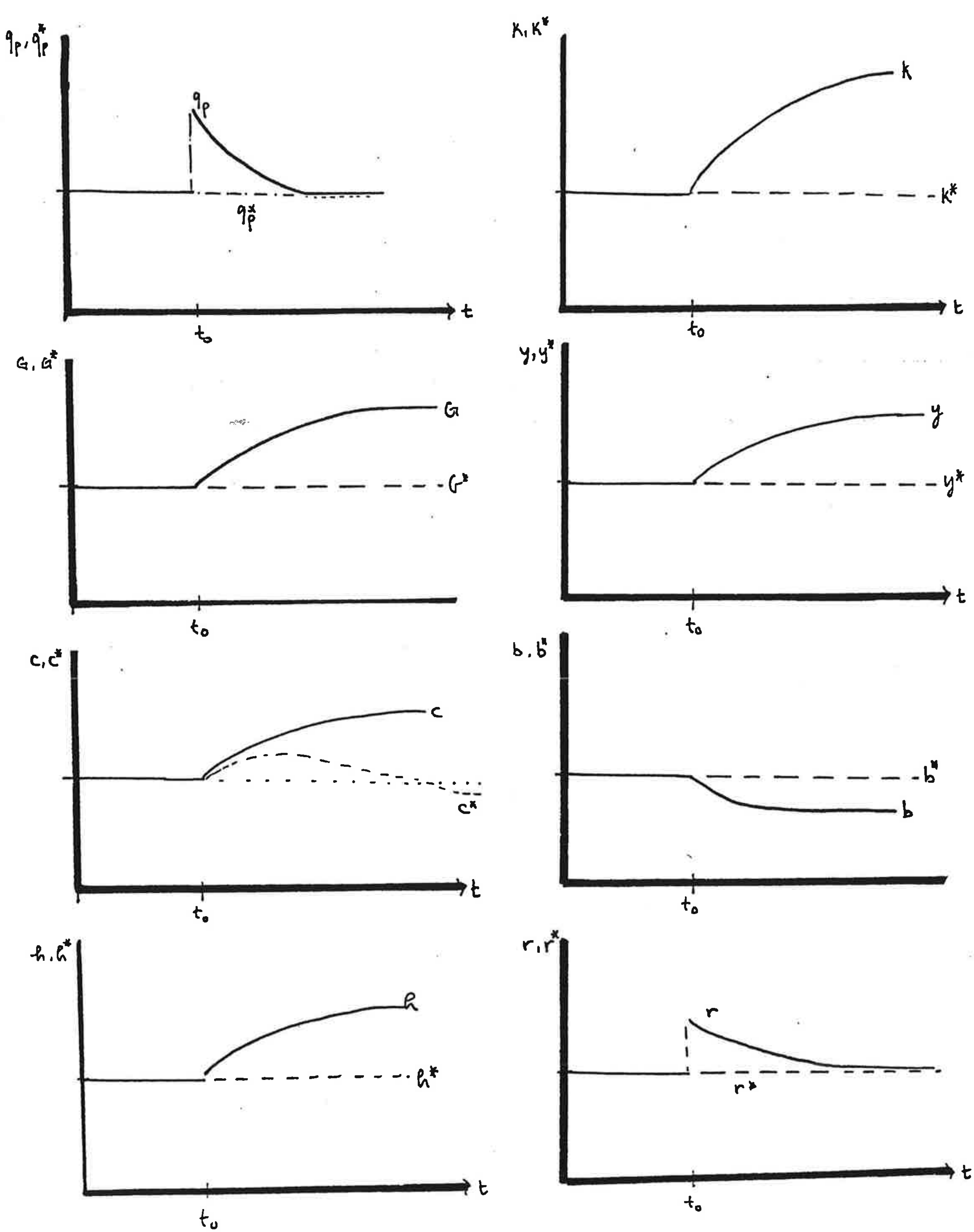


Figure 9 continued

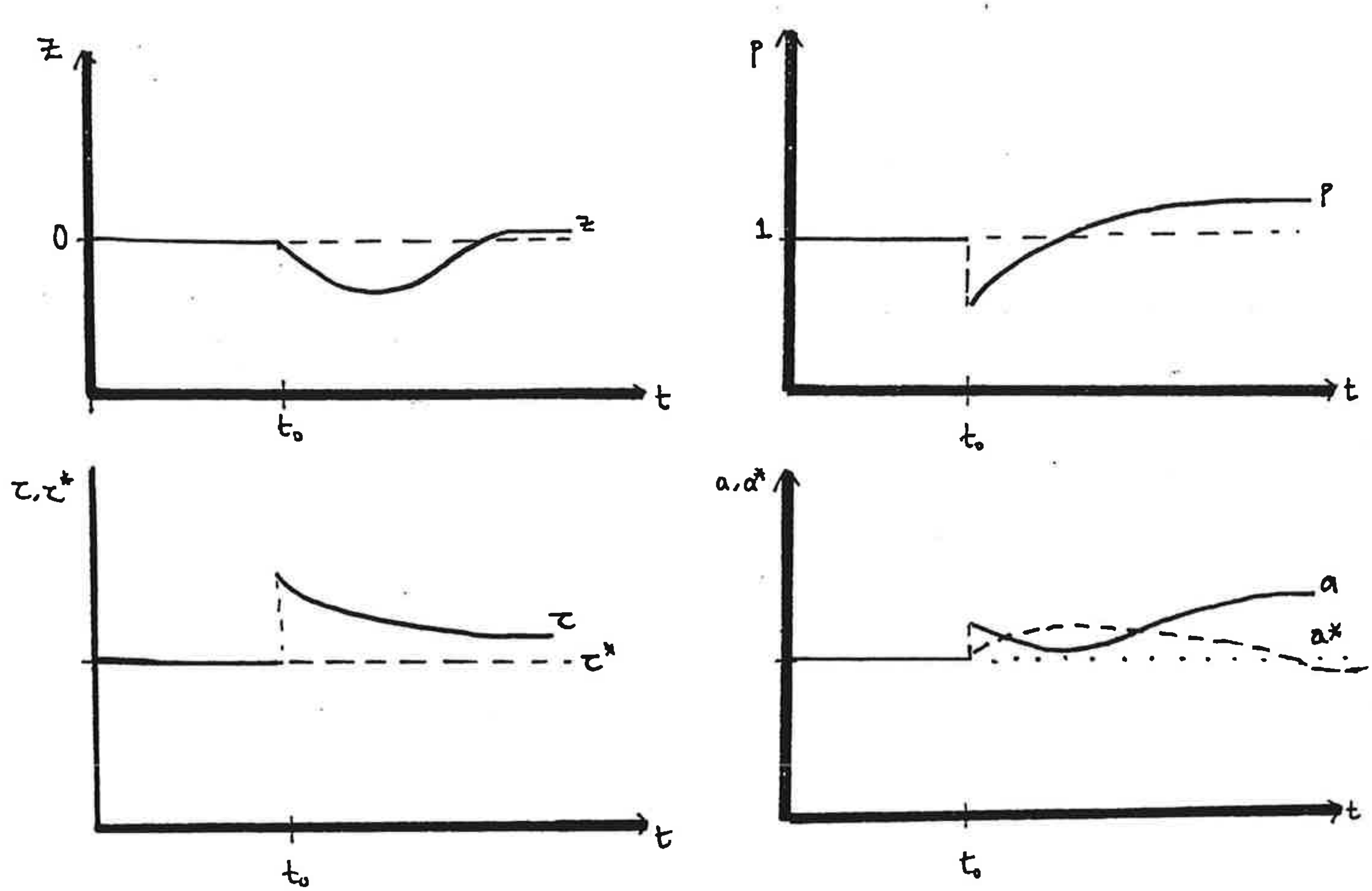


Figure 10: Temporary Balanced Budget Increase In i_g

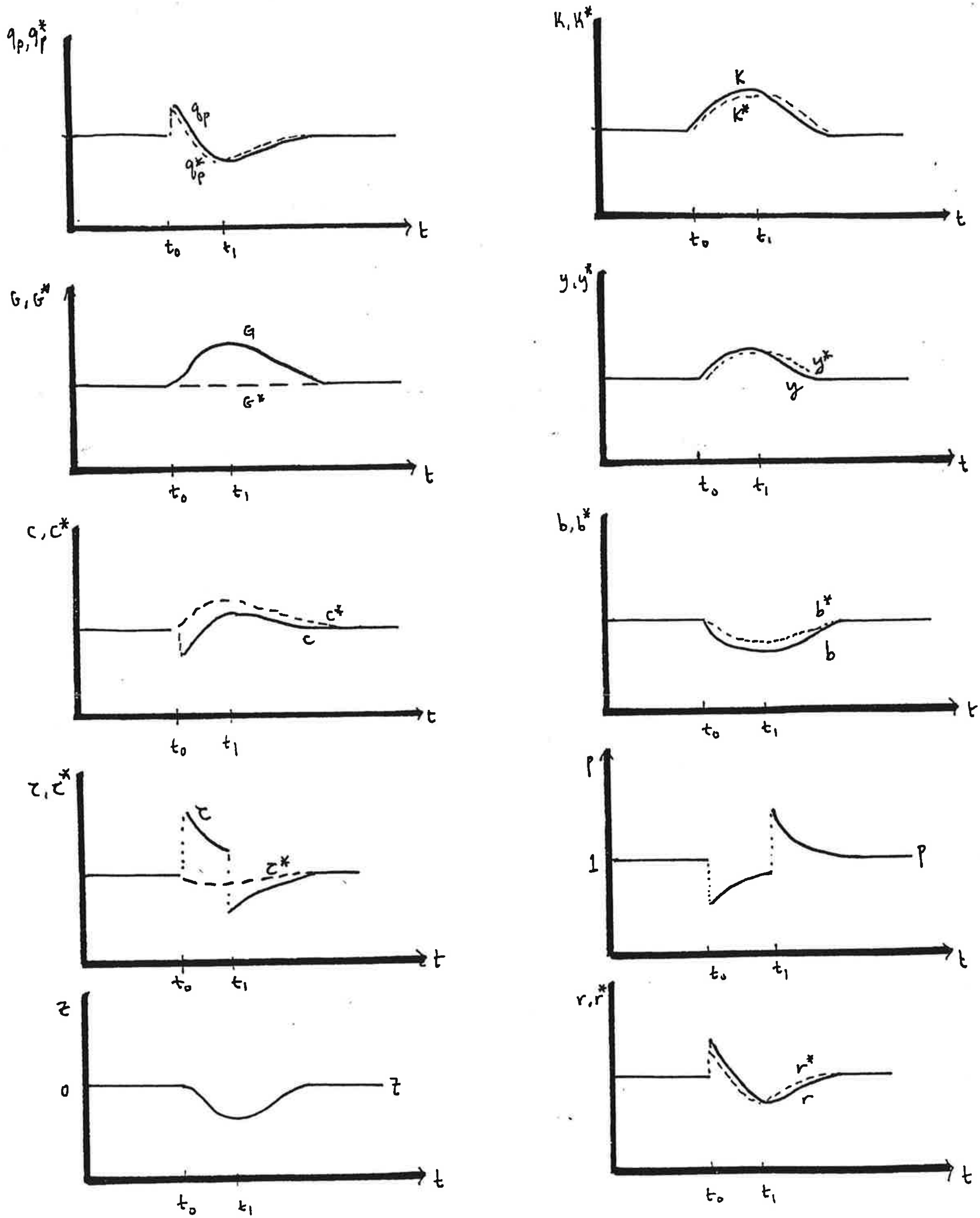


Figure 11: Unanticipated Permanent Deficit-Financed Increase in i_g

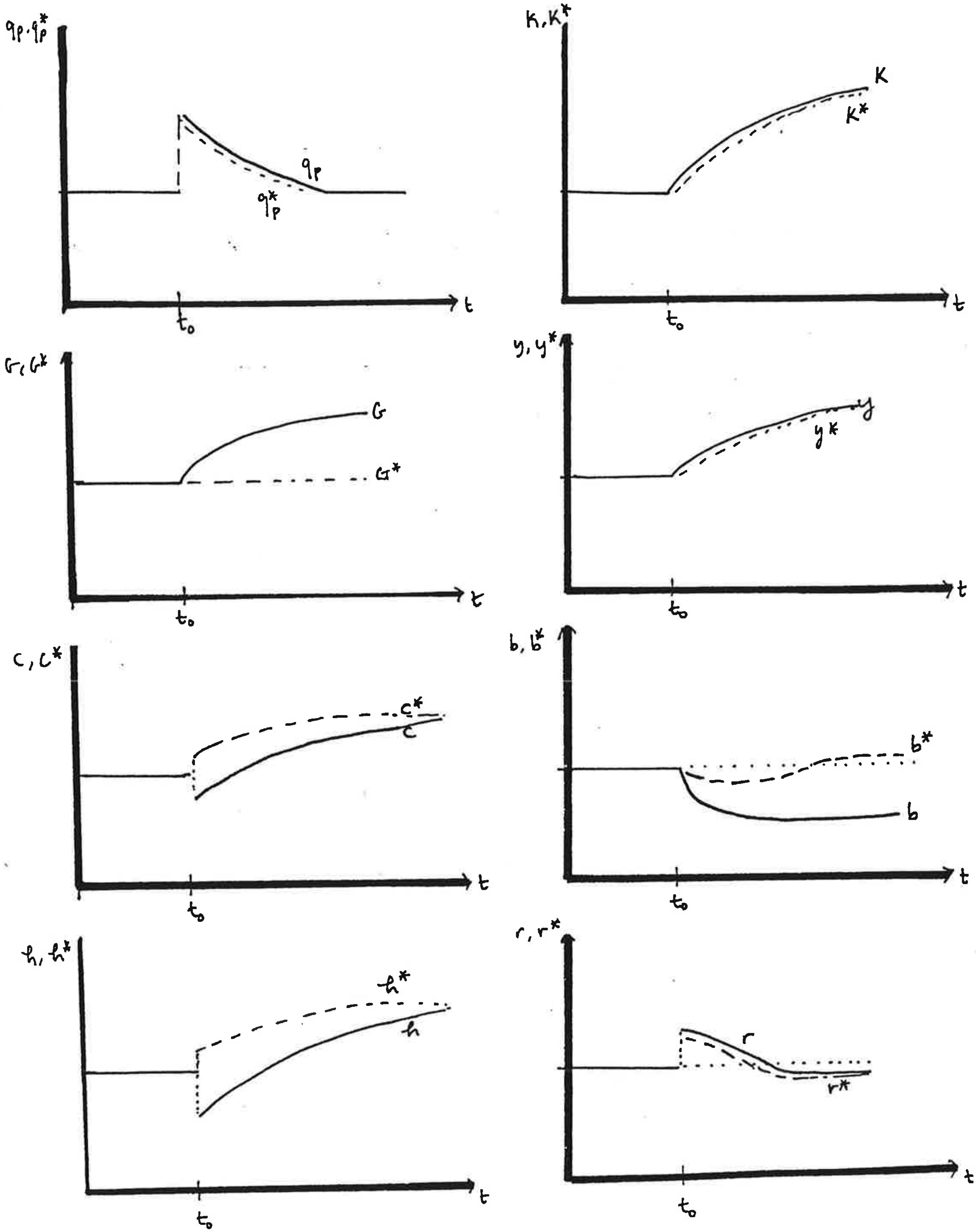


Figure 11 continued

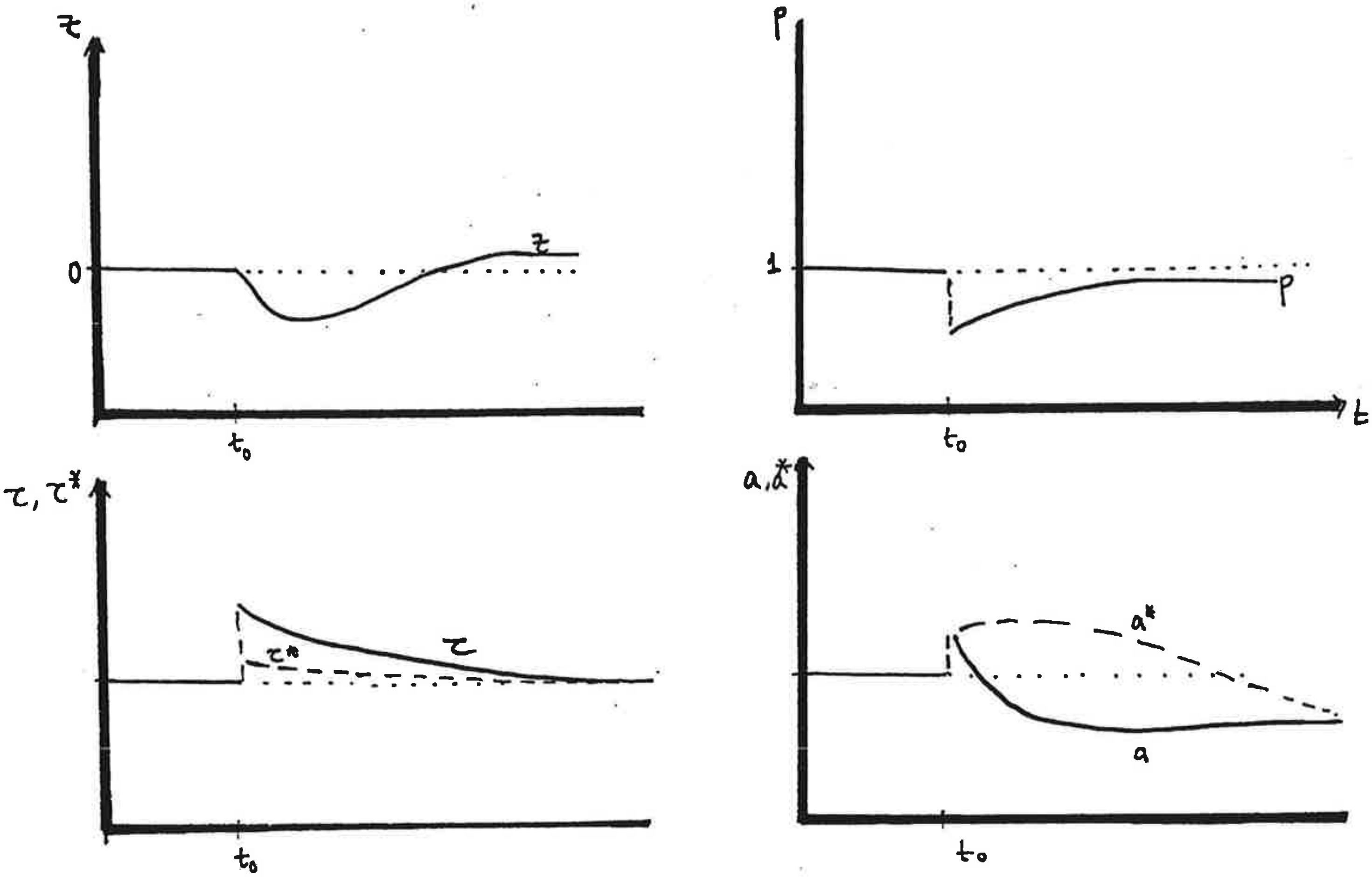


Figure 12: Unanticipated Permanent Balanced Budget Increase in c_g

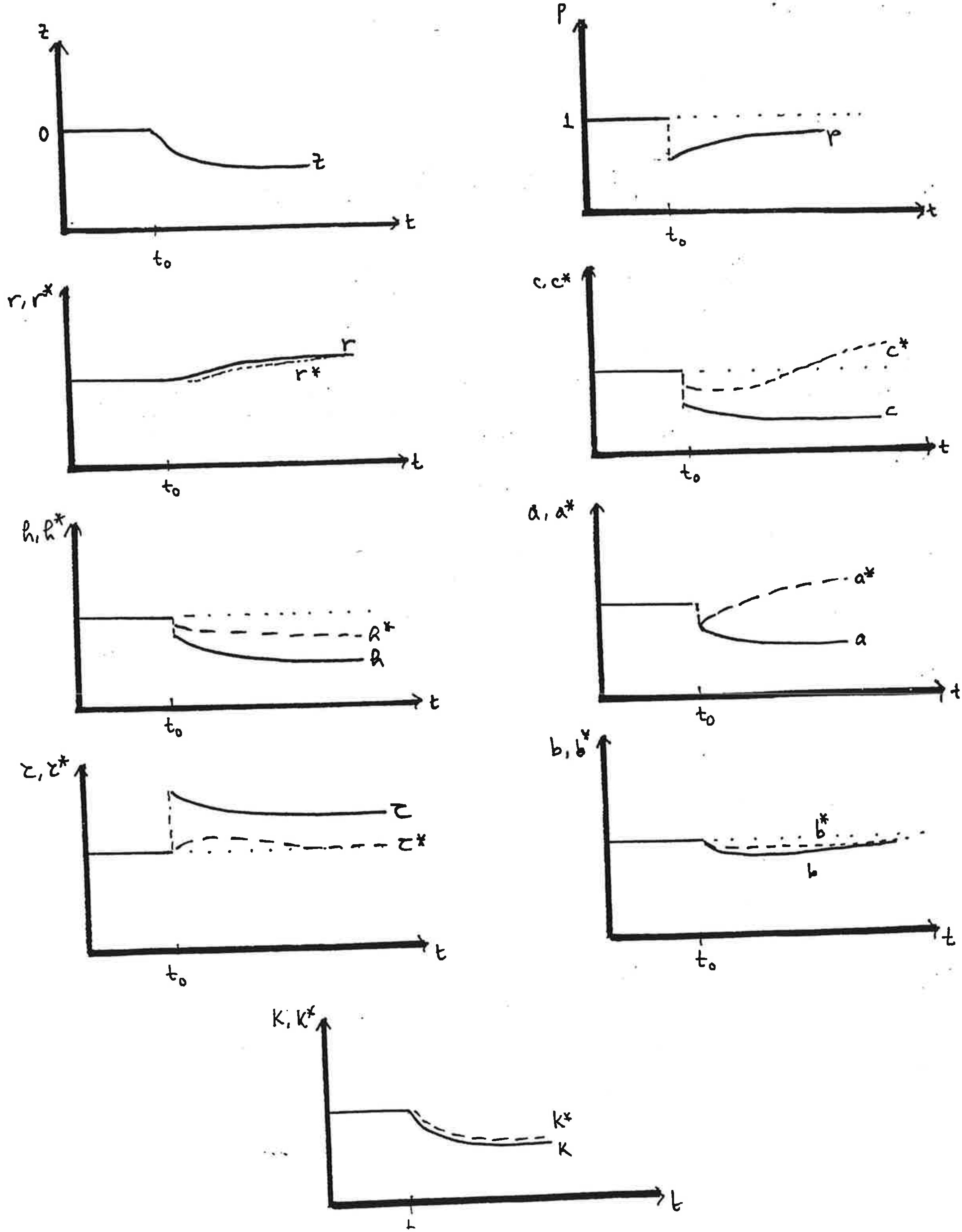


Figure 13: Unanticipated Permanent Deficit-Financed Increase in c_g

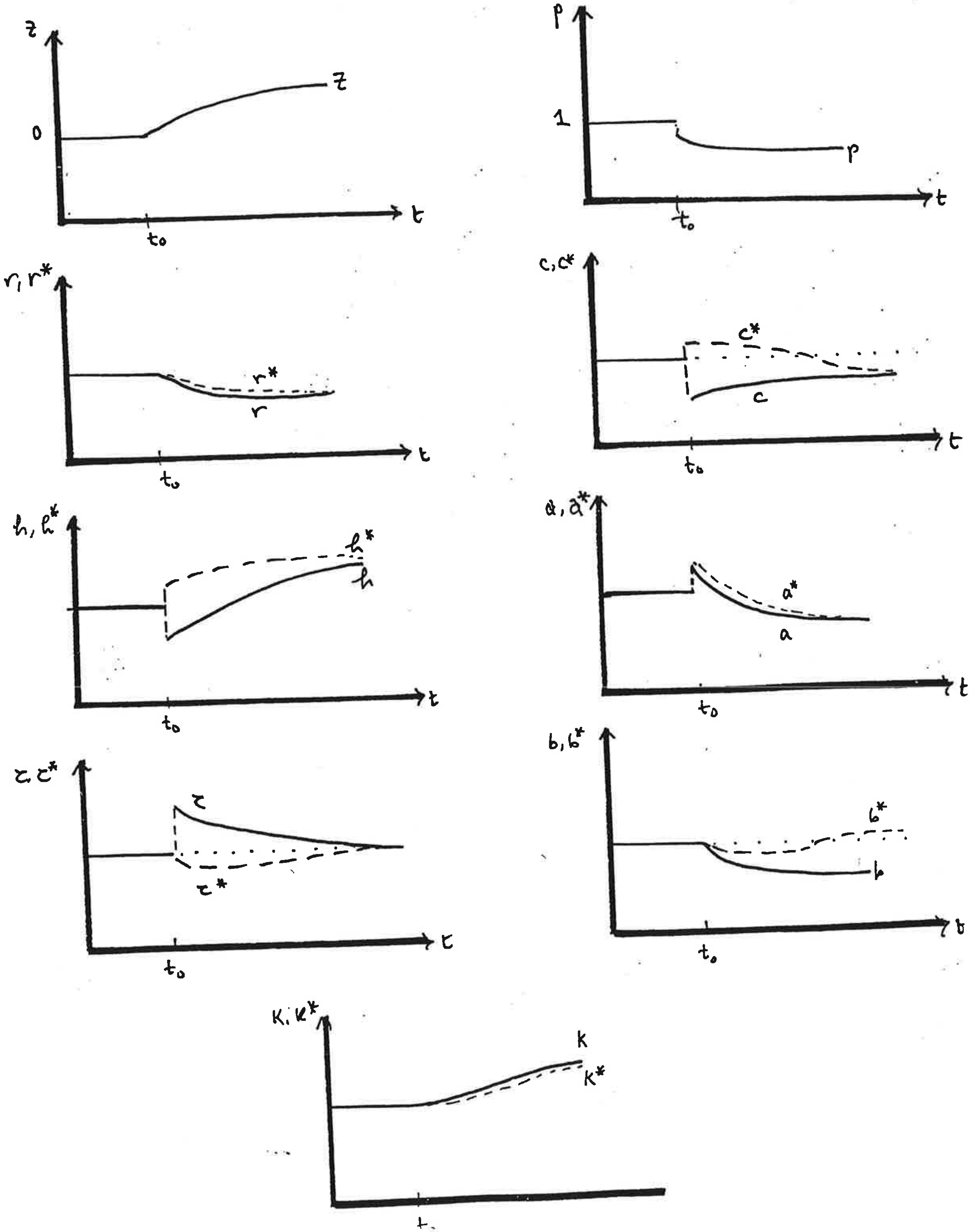


Figure 14: Unanticipated Permanent Increase In Long Run τ_0

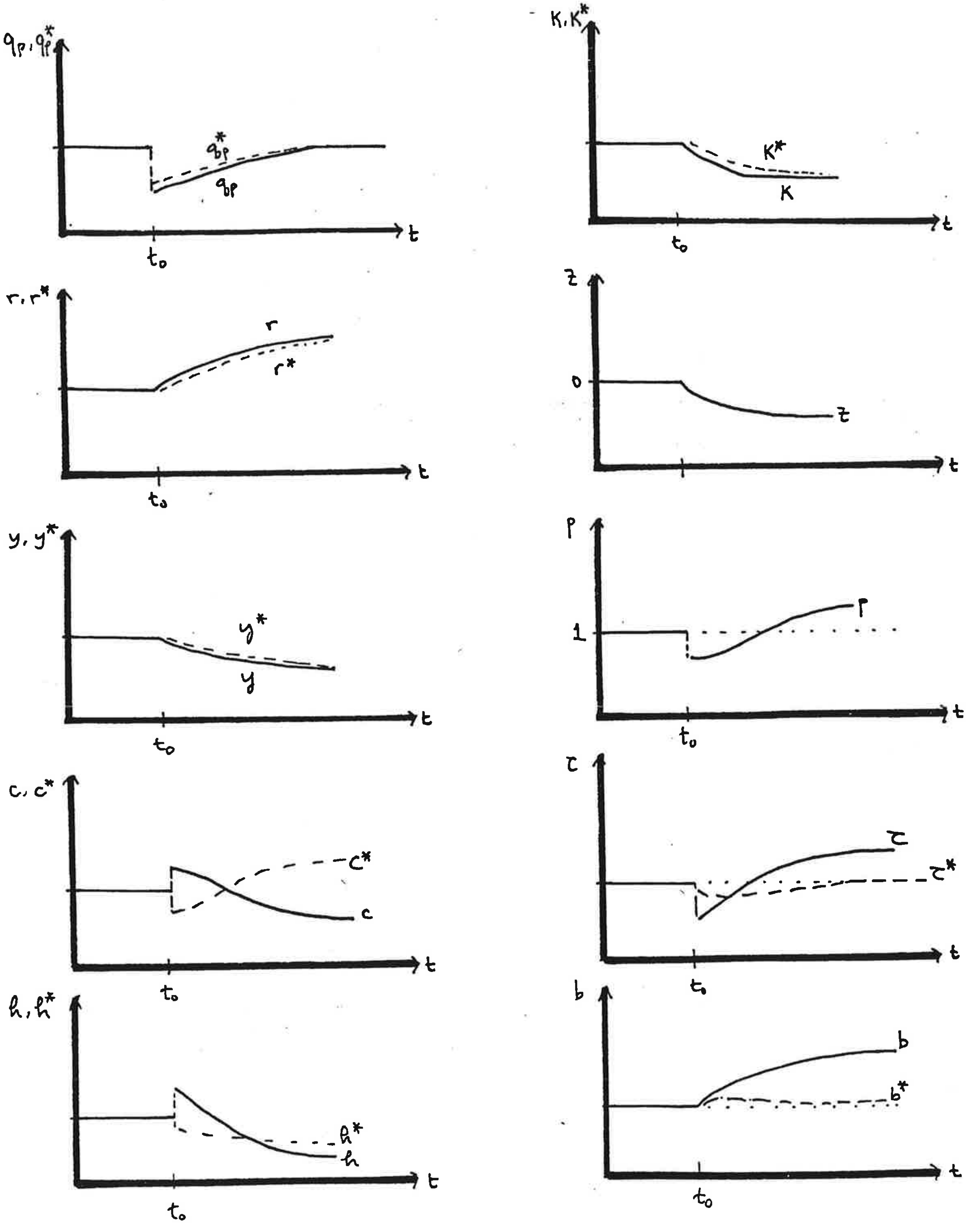


Figure 15: Unanticipated Permanent Decreases in Long Run τ_0 and τ_0^*

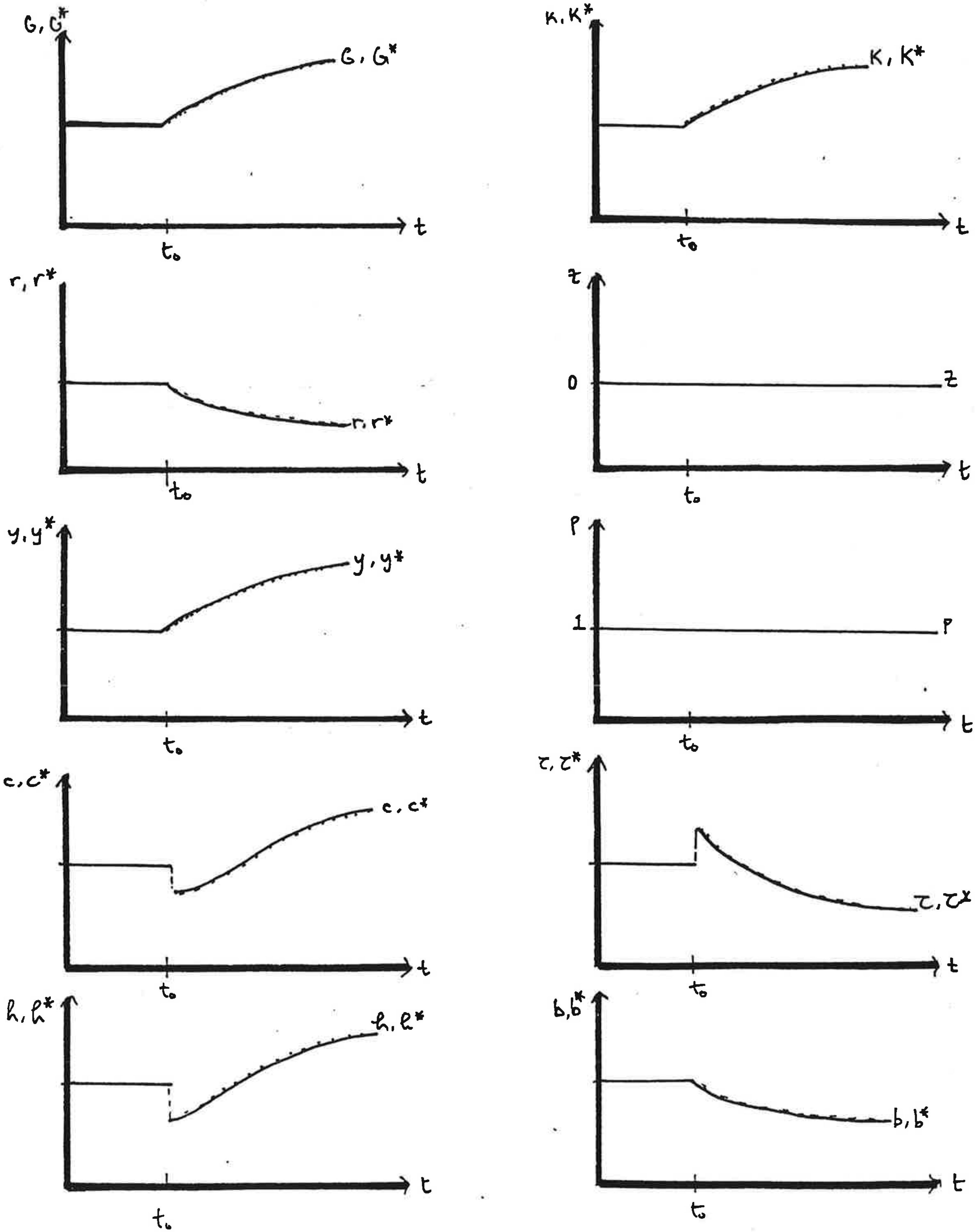


Figure 16: Unanticipated Permanent Long Run Increase in τ_0 and Decrease in τ_0^*

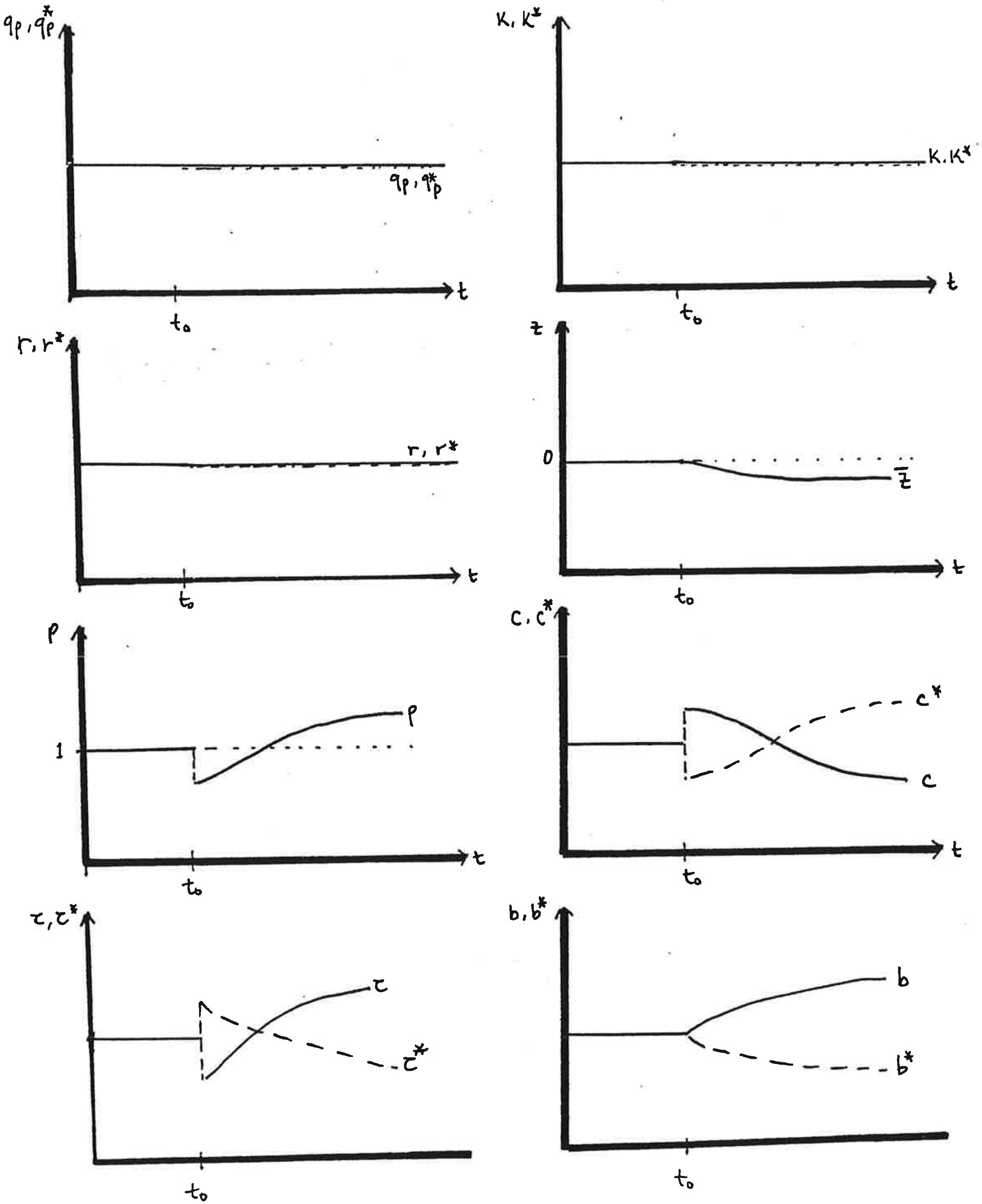


Figure 17: **Unilateral Conversion to Consumption Taxation**

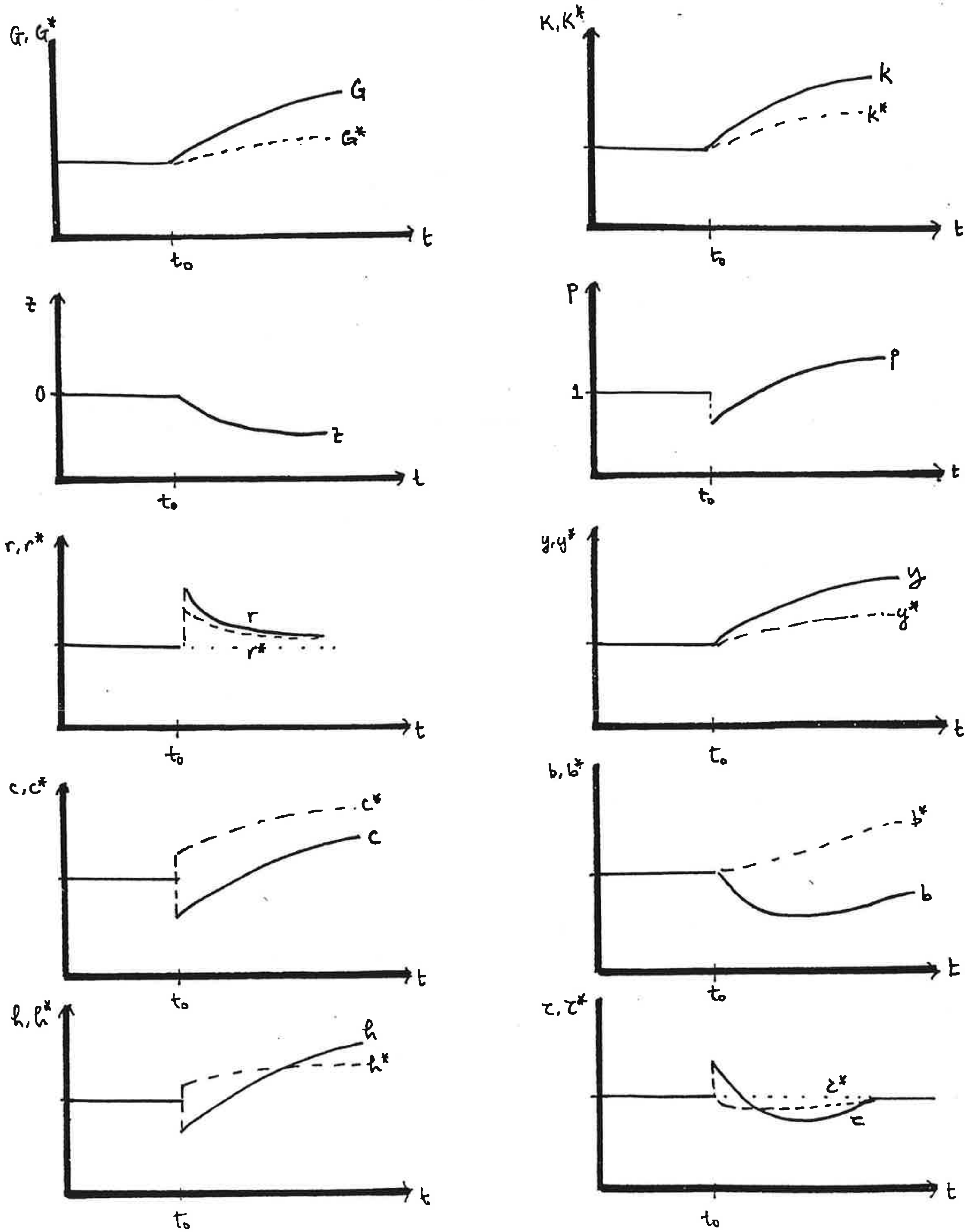


Figure 18: Unilateral Conversion to Capital Income Taxation

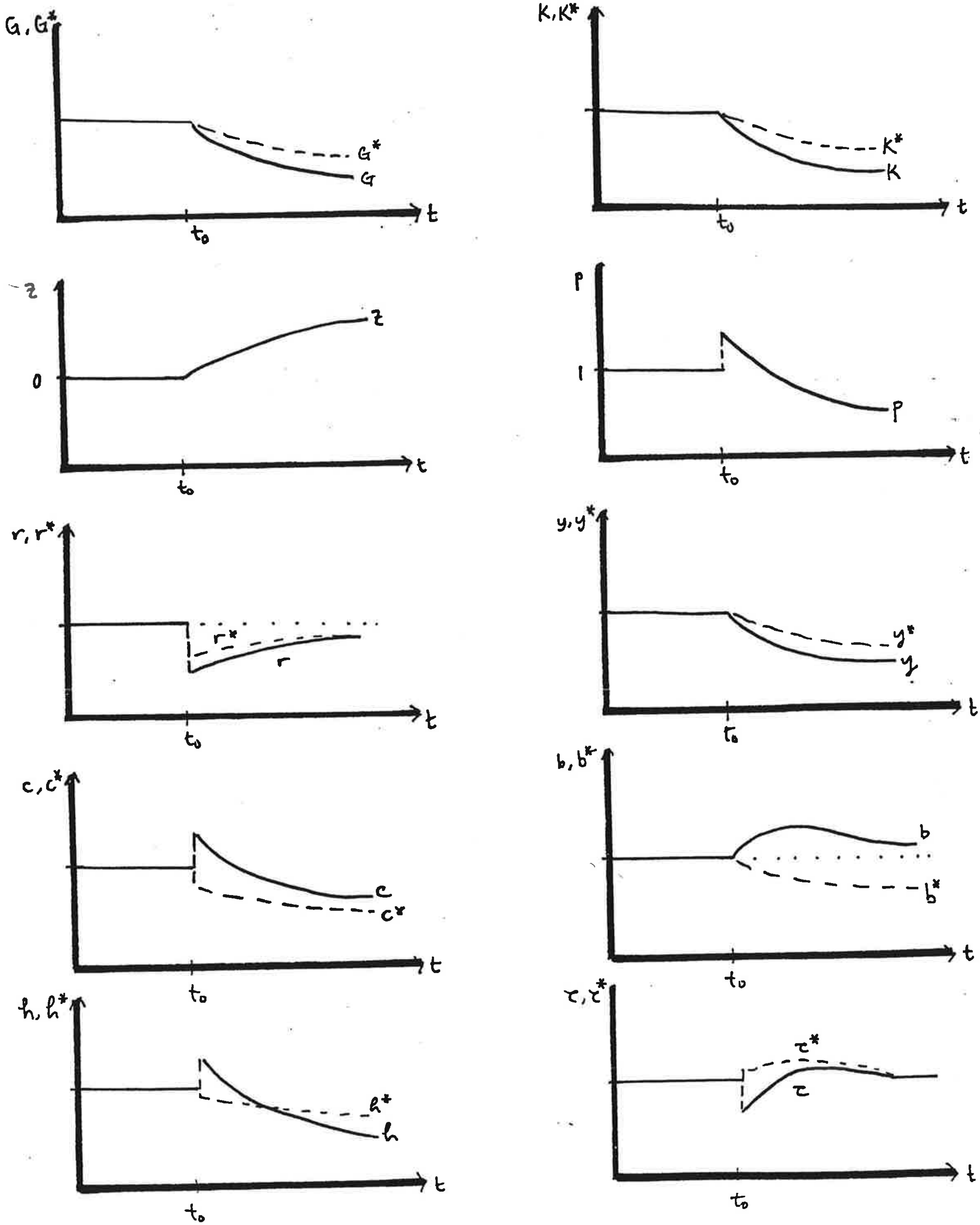


Figure 19: Globally Coordinated Conversion to Consumption Taxation

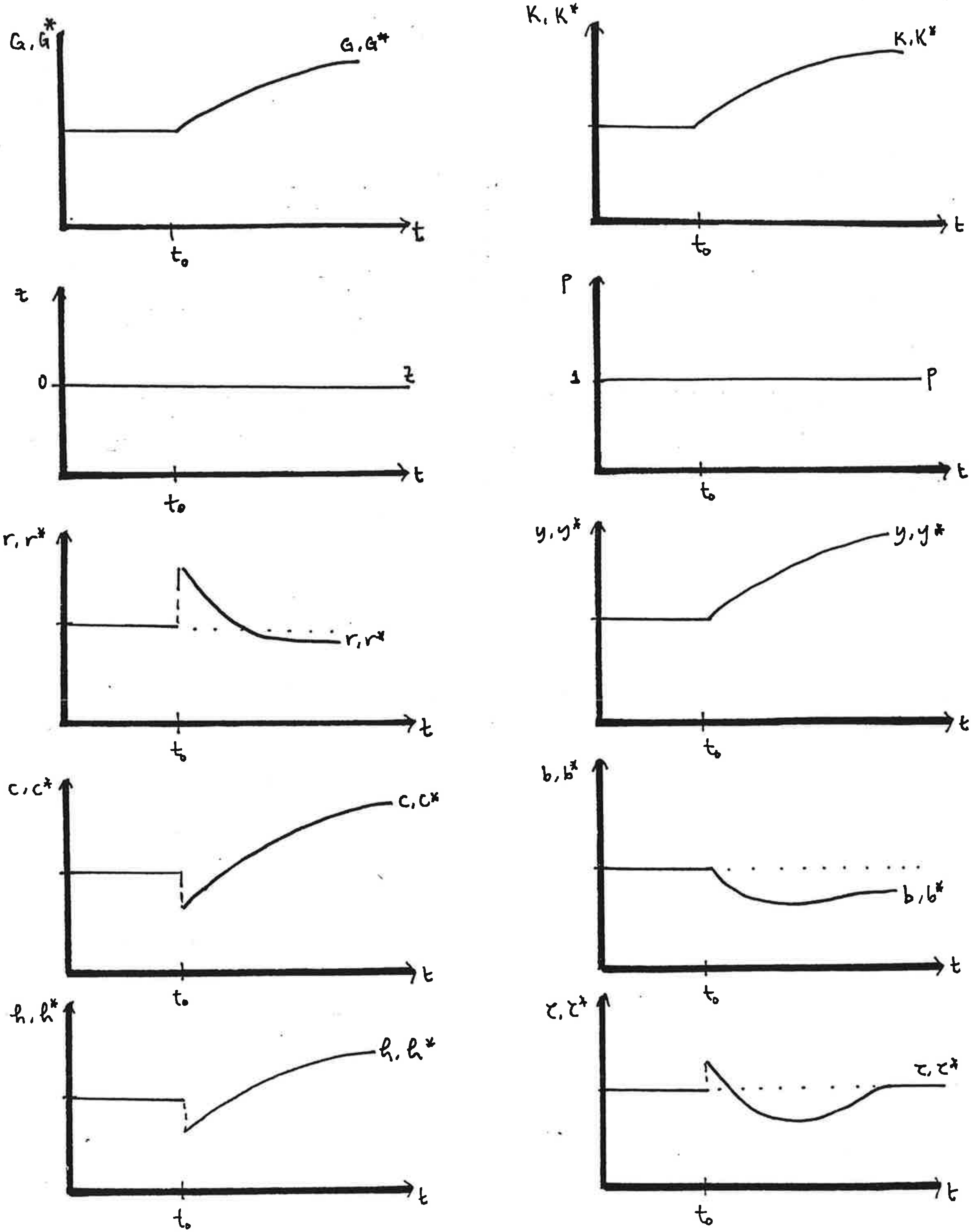


Figure 20: Globally Coordinated Conversion to Capital Income Taxation